Problem A.1 : Journey to Proxima Centauri (4 Points)

The diameter of the Sun is 1.39 million kilometres and the Earth is 8.3 light minutes far away. Proxima Centauri is the nearest star - it has a distance of 4.24 light years to our Sun.

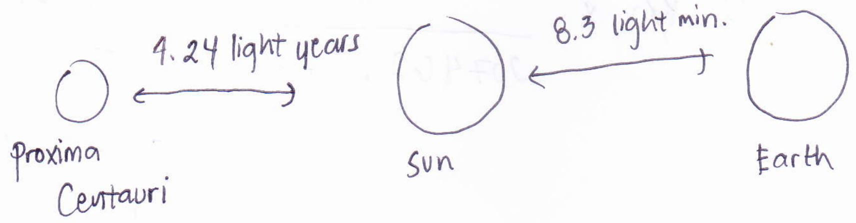
(a) How long does it take to travel to Proxima Centauri with

(i) an airplane (920 km/h) or

(ii) with the *Voyager 1* space probe (17 km/s).

(b) Let the Sun have the size of a tennis ball (diameter: 6.7 cm): How far away is the Earth and how far away is Proxima Centauri on this scale?

a. i.



$4.24 \text{ light } \cancel{\text{years}} \times \frac{9.46 \times 10^{12} \text{ km}}{1 \text{ light } \cancel{\text{year}}} = 4.01104 \times 10^{13} \text{ km}$ → Proxima Centauri's distance in km

$8.3 \text{ light minutes} \times \frac{1.8 \times 10^{7} \text{ km}}{1 \text{ light min.}} = 1.494 \times 10^{8} \text{ km}$ → Earth's distance in km

$4.01104 \times 10^{13} \text{ km} + 1.494 \times 10^{8} \text{ km} = 4.01105494 \times 10^{13} \text{ km}$

$4.01105494 \times 10^{13} \cancel{\text{km}} \times \frac{1 \text{ hour}}{920 \cancel{\text{km}}} = \boxed{4.359842326 \times 10^{10} \text{ hours}}$

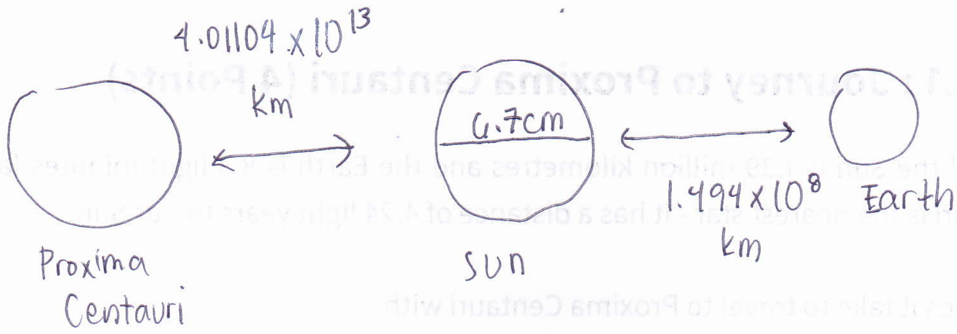
a. i. i

$\frac{17 \text{ km}}{\cancel{s}} \times \frac{3600 \cancel{s}}{1 \text{ hr}} = 61,200 \text{ km/hr}$

$4.01105494 \times 10^{13} \cancel{\text{km}} \times \frac{1 \text{ hr}}{61,200 \cancel{\text{km}}} = \boxed{6.55401134 \times 10^{8} \text{ hours or } 2.359444082 \times 10^{12} \text{ seconds}}$

A.I.

b.



$$\frac{1.39 \times 10^{6} \text{ km}}{6.7 \text{ cm}} = 207462.6866 \text{ km/cm}$$

Proxima Centauri $4.01104 \times 10^{13} \cancel{\text{km}} \times \frac{1 \text{ cm}}{207462.6866 \cancel{\text{km}}} = \boxed{193,337,899.3 \text{ cm}}$

↑ This is how far away Proxima Centauri from the Sun.

Earth $1.494 \times 10^{8} \cancel{\text{km}} \times \frac{1 \text{ cm}}{207462.6866 \cancel{\text{km}}} = \boxed{720.1294963 \text{ cm}}$

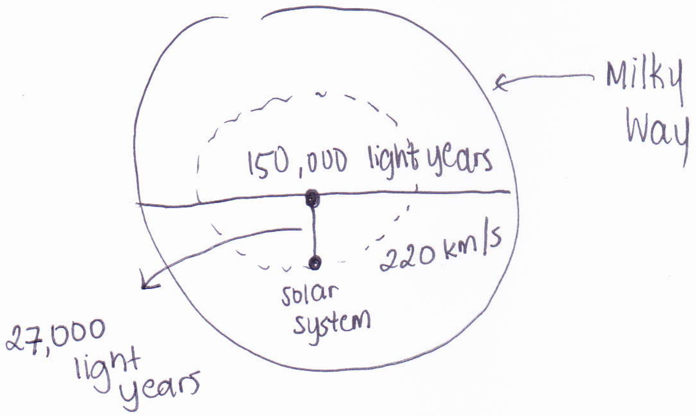
↑ This is how far away Earth from the Sun.

Problem A.2 : Orbit of the Solar System (4 Points)

The Milky Way has a diameter of about 150,000 light years. Our solar system is located 27,000 light years from the center of the Milky Way and orbits the center with a speed of 220 km/s.

(a) How long does it take for the solar system to circle the center of the Milky Way?
(b) The earth has formed about 4.5 billion years ago. How often has the earth circled the center?

ⓐ.



$$27{,}000 \text{ light years} \times \frac{9.46 \times 10^{12} \text{ km}}{1 \text{ light year}} = 2.5542 \times 10^{17} \text{ km}$$

$$\frac{220 \text{ km}}{\cancel{s}} \times \frac{3600 \cancel{s}}{1 \text{ hour}} \times \frac{24 \text{ hours}}{1 \cancel{\text{day}}} \times \frac{365 \cancel{\text{days}}}{1 \text{ year}} = 6{,}937{,}920{,}000 \text{ km/yr}$$

$$\text{Circumference} = 2\pi r = 2\pi (2.5542 \times 10^{17} \text{ km})$$

$$= 1.60485191 \times 10^{18} \text{ km}$$

$$1.60485191 \times 10^{18} \cancel{\text{km}} \times \frac{1 \text{ year}}{6{,}937{,}920{,}000 \cancel{\text{km}}}$$

$$= \boxed{231{,}315{,}897.4 \text{ years}}$$

ⓑ.

$$\frac{4{,}500{,}000{,}000 \cancel{\text{years}}}{231{,}315{,}897.4 \cancel{\text{years}}} = \boxed{19.45391584 \text{ times}}$$

Problem A.3 : Distance to Arcturus (4 Points)

The stellar parallax of the star Arcturus in the constellation Boötes was measures with $0.09''$.

(a) Calculate the distance (in parsec) between Arcturus and the Earth.
(b) How long does it take to send a light message from Earth to Arcturus?

ⓐ

$$1 \text{ parsec} = 206{,}264.81 \text{ AU} = 3.1 \times 10^{16} \text{ m}$$

$$d = \frac{1}{p''} \text{ pc} = \frac{1}{0.09} \text{ pc} = \boxed{11.\overline{11111111} \text{ pc}}$$

ⓑ

$$11.11111111 \text{ pc} \times \frac{3.1 \times 10^{16} \text{ m}}{1 \text{ pc}} = 3.444444444 \times 10^{17} \text{ m}$$

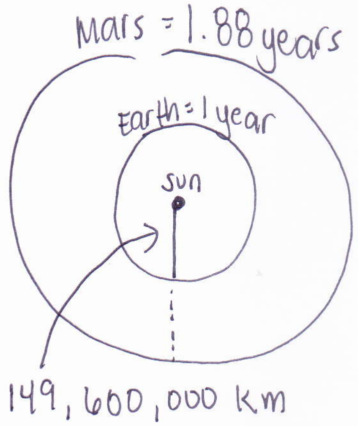
$$3.444444444 \times 10^{17} \text{ m} \times \frac{1 \text{ s}}{3.0 \times 10^{8} \text{ m}} = \boxed{\begin{array}{c} 1{,}148{,}148{,}148 \text{ seconds} \\ \text{or} \\ 36.40753894 \text{ years} \end{array}}$$

Problem A.4 : From Earth to Mars (4 Points)

For a special mission to Mars you need to know the smallest distance between Earth and Mars. However, you have lost your astronomy book and you could only find these values:

Distance Earth to Sun: 149.6 million km
Orbital period Earth: 1.00 years
Orbital period Mars: 1.88 years

By using these values and assuming that Mars and Earth move an circular orbits, calculate the smallest possible distance between Earth and Mars.



$$\left(\frac{T_{Mars}}{T_{Earth}}\right)^2 = \left(\frac{r_{mars}}{r_{Earth}}\right)^3 \rightarrow$$ Kepler's 3rd planetary Law

$$\left(\frac{1.88\text{ yrs}}{1\text{ yr}}\right)^2 = \left(\frac{r_{Mars}}{149,600,000\text{ km}}\right)^3$$

$$r_m = 227,878,574\text{ km}$$

radius of mars − radius of Earth = shortest distance between Earth = Mars

$$227,878,574\text{ km} - 149,600,000\text{ km} = \boxed{78,278,573.97\text{ km}}$$

Problem B.1 : New Star (6 Points)

You have discovered a new star in the Milky Way: Your new star is red and has $3/5$ the temperature of our Sun. The new star emits a total power that is 100,000 times greater than the power emitted by our Sun.

(a) Determine the spectral type (i.e. spectral classification) of the new star.
(b) How many times bigger is the radius of the new star compared to the radius of our Sun?

Sun → Temperature = 5,800 K
Radius = 695, 510 km
Power = 3.846×10^{26} Watts

New Star → Temperature = $\left(\frac{3}{5}\right)$ 5,800K = $\underline{3,480\text{K}}$ → M
Power = $3.846 \times 10^{26} \times 100{,}000 = 3.846 \times 10^{31}$ Watts

ⓐ.

Morgan-Keenam System

O	> 25,000 K
B	10,000 K − 25,000 K
A	7,500 K − 10,000 K
F	6,000 K − 7,500 K
G	5,000 K − 6,000 K
K	3,500K − 5,000 K
* M	< 3,500K

Luminosity Classes

* I	Supergiant
II	Bright Giant
III	Giant
IV	Subgiant
V	Main sequence Dwarf
VI	Subdwarf
VII	White dwarf

Luminosity = 4π Radius^2 σ Temp^4

$$3.846 \times 10^{31} = 4\pi R^2 (5.67 \times 10^{-8}\ \text{Wm}^{-2}\text{K}^{-4})(3{,}480\text{K})^4$$

Radius = $6.066660483 \times 10^{11}$ m

$$b = \frac{L}{4\pi R^2}\ \text{Wm}^{-2} = \frac{3.846 \times 10^{31}\ \text{W}}{4\pi(6.066660483 \times 10^{11}\text{m})^2} = 8{,}315{,}723.389$$

$\boxed{\text{M2I}}$

ⓑ.

$$\frac{6.066660438 \times 10^{11} \cancel{m}}{695,510,000 \cancel{m}} = \boxed{872.2607127 \text{ times}}$$

Problem B.2 : Moon Satellite (6 Points)

The Moon has a mass of $M = 7.3 \cdot 10^{22}\ kg$, a radius of $R = 1.7 \cdot 10^{6}\ m$ and a rotation period of $T = 27.3$ days. Scientists are planning to place a satellite around the Moon that always remains above the same position (geostationary).

(a) Calculate the distance from the Moon's surface to this satellite.
(b) Explain if such a Moon satellite is possible in reality.

$M = 7.3 \times 10^{22}$ kg
$R = 1.7 \times 10^{6}$ m
$T = 27.3 \text{ days} \times \frac{24 \text{ hrs}}{1 \text{ day}} \times \frac{3600 \text{ s}}{1 \text{ hr}} = 2,358,720$ seconds

ⓐ

$$\frac{T^2}{R_e} = \frac{4\pi^2}{GM}$$

Let R_e = distance of the satellite from the Moon's core such that the satellite will not be pulled by the moon.

$$R_e = \sqrt[3]{\frac{T^2 GM}{4\pi^2}} = \sqrt[3]{\frac{(2,358,720)^2(6.67 \times 10^{-11})(7.3 \times 10^{22}\ kg)}{4\pi^2}}$$

$$R_e = 88,202,435.8\ m$$

height $= R_e - R = 88,202,435.8\ m - 1,700,000\ m$

$$= \boxed{86,502,435.8\ m}$$

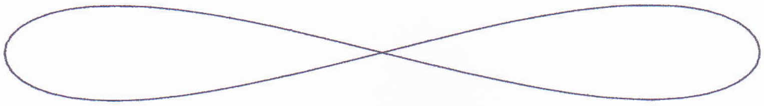
4

ⓑ Creating a satellite for the moon is possible but it will not last long. Soon after its orbit, the satellite will crash due to unstable gravitational field known as "perturbation." The strength of gravity from the Earth tangles with the force of gravity of the Moon especially experienced when the satellite is above 692 km; thus, making the satellite's orbit decay until it crashes.

2

Problem B.3 : Binary Star System (6 Points)

You are the captain of a spaceship that is circling through a binary star system. Due to the gravitational forces and the rocket engines, the orbit of your spaceship looks like that:



The position of your spaceship (in AU) at the time t (in days) is given by:

$$x = 5\sin(t) \qquad y = \sin(2t) \qquad z = 0$$

(a) How long does it take your spaceship to circle the orbit once?
(b) Find an equation that calculates the velocity $v(t)$ of your spaceship at a given time t.
(c) The two stars are positioned at the points $(4, 0, 0)$ and $(-4, 0, 0)$: What is the distance of your spaceship to the stars at the time $t = \frac{\pi}{2}$?

ⓐ $X = 5\sin(t) \qquad y = \sin(2t) \qquad z = 0$

In the form $y = a\sin(bx)$ or $y = a\cos(bx)$

$|a| \rightarrow$ amplitude

$\frac{2\pi}{b} \rightarrow$ Period

Period of $X \rightarrow \frac{2\pi}{1} = 2\pi$ Period of $Y \rightarrow \frac{2\pi}{2} = \pi$

Total period $= 2\pi + \pi = \boxed{3\pi}$

ⓑ $V_x = \frac{dx}{dt} = 5\cos(t)$ $V_y = \frac{dx}{dt} = \cos(2t)$

$$\boxed{V(t) = 5\cos(t) + \cos(2t)}$$

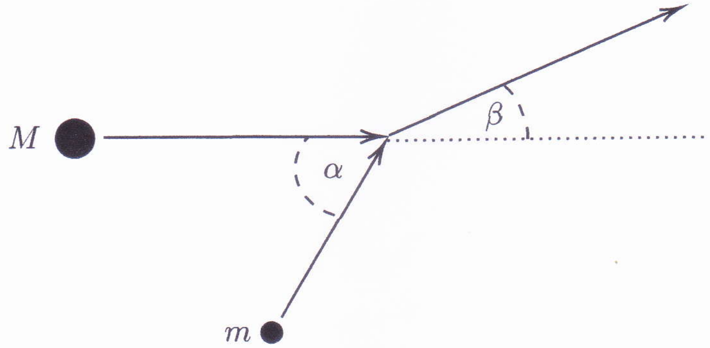
ⓒ distance $= 5\sin(t) + \sin(2t)$

$= 5\sin(\pi/2) + \sin(2\cdot\frac{\pi}{2})$

$= \boxed{0.1918643331 \text{ Au}}$

Problem B.4 : Asteroid Collision (6 Points)

A warning system has calculated that two asteroids will collide not far from Earth any time soon. The smaller asteroid has the mass m and moves with the velocity v_m. The bigger asteroid has the mass $M = 3m$ and the velocity of $v_M = \frac{1}{2}v_m$. They collide at an angle of $\alpha = 60°$ and turn into a single heavy asteroid (inelastic collision):



(a) Calculate the velocity of the single object after the collision.

(b) Determine the angle β after the collision.

Let :

$3m = M$

$V_M = \frac{1}{2} V_m$

ⓐ

	X	Y
M	$3(m)(\frac{1}{2}V_m) = \frac{3}{2} m V_m$	0
m	$m V_m \cos 60° = \frac{1}{2} m V_m$	$m V_m \sin 60° = \frac{\sqrt{3}}{2} m V_m$
M + m	$(M+m) V_f \cos \beta$	$(M+m) V_f \sin \beta$

$$P_{TOT_i} = \sqrt{(2mV_m)^2 + (\tfrac{\sqrt{3}}{2} mV_m)^2} = \frac{\sqrt{19}}{2} m V_m$$

$$P_{TOT_i} = P_{TOT_f}$$

Total initial momentum = Total final mome

$\uparrow$ Law of momentum conservation

$$P_{TOT_f} = \sqrt{(4mV_f \cos\beta)^2 + (4mV_f \sin\beta)^2}$$

$$= \sqrt{16m^2V_f^2\cos^2\beta + 16m^2V_f^2\sin^2\beta}$$

$$\left(\frac{\sqrt{19}}{2} mV_m\right)^2 = \left(\sqrt{16m^2V_f^2\cos^2\beta + 16m^2V_f^2\sin^2\beta}\right)^2$$

$$\frac{19}{4}\cancel{m}^2 V_m{}^2 = 16\cancel{m^2} V_f{}^2 \cos^2\beta + 16\cancel{m^2} V_f{}^2 \sin\beta$$

$$\frac{19}{4} V_m{}^2 = 16 V_f{}^2 \left(\overset{1}{\cancel{\cos^2\beta + \sin^2\beta}}\right)$$

$$\sqrt{\frac{19}{64} V_m{}^2} = \sqrt{V_f{}^2}$$

$$\boxed{\frac{\sqrt{19}}{8} V_m = V_f}$$

ⓑ.

$$\beta = \tan^{-1}\left(\frac{P_{y\,TOT}}{P_{x\,TOT}}\right) = \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{2}\right) = \boxed{23.41^\circ \text{ N of Earth}}$$

Problem C.1 : The Sunburst Arc (10 Points)

This problem requires you to read following recently published scientific article:

The Sunburst Arc: Direct Lyman α escape observed in the brightest known lensed galaxy.
T.E. Rivera-Thorsen, H. Dahle, M. Gronke, M. Bayliss, J.R. Rigby, R. Simcoe, R. Bordoloi, M. Turner, and G. Furesz,
Astronomy & Astrophysics 608, (2017). Link: https://iaac.space/doc/Pre-Final-Article-2017-Sunburst-Arc.pdf

Answer following questions related to this article:

(a) Why is it difficulty for *LyC radiation* to escape galaxies with high star formation rates?

It contains large amounts of neutral hydrogen which obstructs the escape of LyC radiation.

(b) What is the difference between the *density-bounded medium* and the *picket fence model*?

Density bounded mediums are surrounded by isotropic gas with low column density of neutral hydrogen which, conveniently does not completely depreciate the amount of escaping LyC radiation. Whilst, the picket fence model depicts the scenario wherein neutral hydrogen is high in column density but does not entirely block the ionizing sources that enable LyC radiation to escape.

(c) Explain the spectral shape of the *perforated shell model* (see article: figure 1, right box).

In this model, Lyα line shape narrows on its central peak that escapes through uncovered sight lines; which is superimposed on the characteristic double-peak profile that emerges from optically thick gas.

(d) What is the *Sunburst Arc* and how was it discovered?

It is a lensed galaxy that contains a direct escape for Lyman α photons through a perforated medium. It was reported by Dahle et. al and was discovered by follow-up imaging of its lensing cluster through its Sunyaev-Zel'dovich effect in Planck data.

(e) When did the scientist observe the object and which instruments did they use?

The scientists observed the lensed galaxy on the 24th of May 2017 using the Magellan Echelette (MagE) spectograph attached on the Magellan-I (Blade) telescope.

(f) What is the redshift of the *Sunburst Arc* and how was it determined from the data?

The scientists determined the redshift of the Sunburst Arc based on the Folded-port Infrared Echelle (FIRE) spectrum by fitting a single Gaussian profile to each of the four strong emission line H Beta and H Alpha.

(g) Explain the difference between the right and the left diagram in figure 4 (see article).

It shows the comparison of the shape of the central peak between H Alpha and Lyα when exposed to amounts of neutral hydrogen. When significants amounts of neutral hydrogen is exposed, the Lyα line broadened by frequency diffusion. Whilst the H Alpha is narrower suggesting that it has a stronger interaction with neutral gas than Lyα.

Problem C.2 : Dark Matter (10 Points)

This problem requires you to read following recently published scientific article:

Probing Dark Matter Using Precision Measurements of Stellar Accelerations.
A. Ravi, N. Langellier, D.F. Phillips, M. Buschmann, B.R. Safdi, R.L. Walsworth,
arXiv:1812.07578, (2018). Link: https://iaac.space/doc/Pre-Final-Article-2018-1812.07578.pdf

Answer following questions related to this article:

(a) What are the current methods to determine the dark matter density / radial velocity and what are the disadvantages of these methods? It is determined from the Galactic rotation curve, measured through Doppler shifts, or the dispersion of local stellar velocities in the vertical direction about the Galactic mid-plane. measured using astrometry. The problem as to why these methods are not advisably used is that both use the assumption of equilibrium that dynamics have reached steady-state.

(b) Explain the new method for measuring dark matter density that is proposed in the article. They used the same precision radial velocity measurements of the acceleration of individual stars in measuring the Dark Matter density distribution. They highlighted the work of Silverwood and Easther wherein they also propose using stellar accelerations to map the Galactic gravitational field.

(c) Let $a_r(r)$ be the dark matter contribution to the acceleration: Show that the dark matter density is given by $\rho_{DM} \approx \frac{1}{4\pi G}\left(2(A-B)^2 - \frac{\partial a_r}{\partial r}\right)$ with Oort constants $2(A-B)^2 = 2GM(r)/r^3$.

$$\rho_{DM} \approx \frac{1}{4\pi G}\left(2(A-B)^2 - \frac{\partial ar}{\partial r}\right)$$

Let $2(A-B)^2 = 2GM(r)/r^3$

$$\boxed{\rho_{DM} \approx \frac{1}{4\pi G}\left(2GM(r)/r^3 - \frac{\partial ar}{\partial r}\right)}$$

(d) How much does the velocity of the stars change during the lifetime of a human (80 years)?

10 years = 5cm/sec stellar velocity change

$$8 \quad \frac{80~\cancel{\text{years}}}{10~\cancel{\text{years}}} = \frac{x}{5\text{cm/s}} \quad 5\left(8 = \frac{x}{\cancel{5}\text{cm/s}}\right)\cancel{5}\text{cm/s} \quad \boxed{40\text{ cm/s}}$$

(e) What is important for measuring stellar accelerations and which instruments can be used? It is important to measure stellar acceleration because it will help us determine the Radial Velocity (RV) or Dark Matter (DM) density over several years. The instruments ideal for this task are known as "astro-combs" or may be referred to as GPS-disciplined atomic clocks.

(f) Explain the curves of the four diagrams in figure 2 (see article).

The diagrams on Figure 2 show the 4 contributing mechanisms of a synthesized Radial Velocity time series for a single primary star. The curves per graph of Fig. 2 show the effects of acceleration due to the Milky Way gravitational potential, stellar companions, planets and "noise" including stellar magnetic activity.