Problem A.1 : Journey to Proxima Centauri (4 Points)

The diameter of the Sun is 1.39 million kilometres and the Earth is 8.3 light minutes far away. Proxima Centauri is the nearest star - it has a distance of 4.24 light years to our Sun.

(a) How long does it take to travel to Proxima Centauri with
  (i) an airplane (920 km/h) or
  (ii) with the *Voyager 1* space probe (17 km/s).

(b) Let the Sun have the size of a tennis ball (diameter: 6.7 cm): How far away is the Earth and how far away is Proxima Centauri on this scale?

(a) $c = \frac{\Delta S}{\Delta t} \Rightarrow \Delta S = c \cdot \Delta t \Rightarrow 1\,ly = 3 \cdot 10^{8} \cdot 365.25 \cdot 24 \cdot 60 \cdot 60$

$1\,ly \cong 9.467 \cdot 10^{15}\,m = 9.467 \cdot 10^{12}\,Km$

$4.24\,ly \cong 4.014 \cdot 10^{13}\,Km$

(i) $\frac{\Delta S_1}{V_1} = \Delta t_1 = \frac{4.014 \cdot 10^{13}}{920} = 4.36 \cdot 10^{10}\,h \Rightarrow \boxed{\Delta t_1 \cong 4.98 \cdot 10^{6}\ \text{years}}$

(ii) $\frac{\Delta S_2}{V_2} = \Delta t_2 = \frac{4.014 \cdot 10^{13}}{17} = 2.36 \cdot 10^{12}\,s \Rightarrow \boxed{\Delta t_2 \cong 7.48 \cdot 10^{4}\ \text{years}}$

$\Delta S_1 \cong \Delta S_2$

(b) $\frac{1.39 \cdot 10^{6} \cdot 10^{5}}{6.7} = \frac{4.24}{d} \Rightarrow d \cong 2.044 \cdot 10^{-10}\,ly$

$\boxed{d \cong 1935\,Km}$

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Problem A.2 : Orbit of the Solar System (4 Points)

The Milky Way has a diameter of about 150,000 light years. Our solar system is located 27,000 light years from the center of the Milky Way and orbits the center with a speed of 220 km/s.

(a) How long does it take for the solar system to circle the center of the Milky Way?
(b) The earth has formed about 4.5 billion years ago. How often has the earth circled the center?

(a) $\Delta S = 27000 \cdot 2\pi \cong 1.696 \cdot 10^{5}\, ly \Rightarrow \Delta S \cong 1.606 \cdot 10^{18}\, Km$

$\Delta t = \frac{\Delta S}{V} = \frac{1.606 \cdot 10^{18}}{220} \Rightarrow \Delta t = 7.3 \cdot 10^{15}\, s \Rightarrow \boxed{\Delta t \cong 2.313 \cdot 10^{8}\ years}$

(b) $\left.\begin{matrix} \Delta t - 1 \\ 4.5 \cdot 10^{9} - n \end{matrix}\right\} \; n = \frac{4.5 \cdot 10^{9}}{2.313 \cdot 10^{8}} \cong 19.45$ $\boxed{19\ times}$

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Problem A.3 : Distance to Arcturus (4 Points)

The stellar parallax of the star Arcturus in the constellation Boötes was measures with $0.09''$.

(a) Calculate the distance (in parsec) between Arcturus and the Earth.

(b) How long does it take to send a light message from Earth to Arcturus?

(a) $d(pc) = \frac{1}{\pi('')} \Rightarrow d = \frac{1}{0.09} \Rightarrow \boxed{d \cong 11.11 pc}$

(b) Assuming they are at rest in relation to the other.

$d = 11.11 \cdot 3.26 = 36.219 ly \qquad V = c$

$\therefore \boxed{\Delta t \cong 36.22 \text{ years}}$

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Problem A.4 : From Earth to Mars (4 Points)

For a special mission to Mars you need to know the smallest distance between Earth and Mars. However, you have lost your astronomy book and you could only find these values:

Distance Earth to Sun: 149.6 million km
Orbital period Earth: 1.00 years
Orbital period Mars: 1.88 years

By using these values and assuming that Mars and Earth move an circular orbits, calculate the smallest possible distance between Earth and Mars.

$$\frac{P_E^2}{a_E^3} = \frac{P_M^2}{a_M^3} \Rightarrow a_M^3 = P_M^2 \qquad a_E = 1\ U.A.$$

$$a_M = P_M^{\frac{2}{3}} \Rightarrow a_M = 1.523\ U.A.$$

d_{min} will occur when they are in opposition:

$$d_{min} = a_M - a_E = 0.523\ U.A. \qquad 1\ U.A. = 149.6 \cdot 10^6\ Km$$

$$\boxed{\therefore\ d_{min} \cong 78.24 \text{ million Km}}$$

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Problem B.1 : New Star (6 Points)

You have discovered a new star in the Milky Way: Your new star is red and has $3/5$ the temperature of our Sun. The new star emits a total power that is 100,000 times greater than the power emitted by our Sun.

(a) Determine the spectral type (i.e. spectral classification) of the new star.
(b) How many times bigger is the radius of the new star compared to the radius of our Sun?

(a) By the HR diagram: $T_\odot = 5800K \Rightarrow T_{NE} = 3480K$

The classification is M, more especificaly $\boxed{M2}$.

(b) $$\frac{L_{NE}}{L_\odot} = \frac{\cancel{4\pi} \cdot R_{NE}^2 \cdot \cancel{\sigma} \cdot T_{NE}^4}{\cancel{4\pi} \cdot R_\odot^2 \cdot \cancel{\sigma} \cdot T_\odot^4} \Rightarrow \frac{R_{NE}}{R_\odot} = \left(\frac{T_\odot}{T_{NE}}\right)^2 \cdot \sqrt{\frac{L_{NE}}{L_\odot}} = \left(\frac{5}{3}\right)^2 \cdot \sqrt{10^5}$$

$$\Rightarrow \boxed{R_{NE} \cong 878.41 \cdot R_\odot}$$

Is 878.41 times bigger.

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Problem B.2 : Moon Satellite (6 Points)

The Moon has a mass of $M = 7.3 \cdot 10^{22}\ kg$, a radius of $R = 1.7 \cdot 10^{6}\ m$ and a rotation period of $T = 27.3$ days. Scientists are planning to place a satellite around the Moon that always remains above the same position (geostationary).

(a) Calculate the distance from the Moon's surface to this satellite.
(b) Explain if such a Moon satellite is possible in reality.

(a) $\omega = \dfrac{2\pi}{27.3 \cdot 24 \cdot 60 \cdot 60} \cong 2{,}664 \cdot 10^{-6}\ rad/s$

$\omega = \sqrt{\dfrac{GM}{R_S^3}} \Rightarrow R_S = \sqrt[3]{\dfrac{GM}{\omega^2}} \Rightarrow R_S \cong 8.82 \cdot 10^{7} m \cong 88198\ Km$

$h = R_S - R \Rightarrow \boxed{h \cong 86498\ Km}$

(b) $F_E = F_M$

$\dfrac{\cancel{G} \cdot M_E \cdot \cancel{m}}{r_{SE}^2} = \dfrac{\cancel{G} \cdot M_M \cdot \cancel{m}}{r_{SM}^2} = \dfrac{\cancel{G} \cdot M_E \cdot \cancel{m}}{(d - r_{SM})^2} \Rightarrow \dfrac{d}{r_{SM}} - 1 = \sqrt{\dfrac{M_E}{M_M}}$

$\Rightarrow r_{SM} = \dfrac{d}{\sqrt{\frac{M_E}{M_M}} + 1} \Rightarrow r_{SM} \cong 38188\ Km$

$M_M = 7.3 \cdot 10^{22} Kg$
$M_E = 6 \cdot 10^{24} Kg$
$d = 384\,400\ Km$

Since $R_S > r_{SM}$, the satellite would (~~probb~~) probably be attracted to Earth.

$\boxed{\text{It is not possible in reality.}}$

$T = P$

$\dfrac{P^2}{d^3} = \dfrac{4\pi^2}{G(M_E + M_M)} \Rightarrow M_E = \dfrac{4\pi^2 d^3}{G \cdot T^2} - M_M \Rightarrow M_E \cong 5.97 \cdot 10^{24} Kg$

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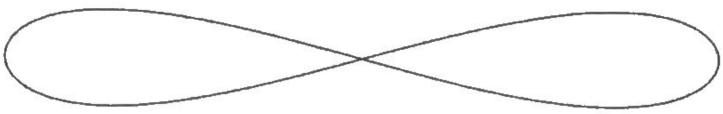
Problem B.3 : Binary Star System (6 Points)

You are the captain of a spaceship that is circling through a binary star system. Due to the gravitational forces and the rocket engines, the orbit of your spaceship looks like that:



The position of your spaceship (in AU) at the time t (in days) is given by:

$$x = 5\sin(t) \qquad y = \sin(2t) \qquad z = 0$$

(a) How long does it take your spaceship to circle the orbit once?
(b) Find an equation that calculates the velocity $v(t)$ of your spaceship at a given time t.
(c) The two stars are positioned at the points $(4, 0, 0)$ and $(-4, 0, 0)$: What is the distance of your spaceship to the stars at the time $t = \frac{\pi}{2}$?

(a) $x = 5\sin(t)$ $\quad t = \frac{\pi}{2} \Rightarrow x = 5, y = 0$ $\quad t = \pi \Rightarrow x = 0, y = 0$

$y = \sin(2t)$

$t = \frac{3\pi}{2} \Rightarrow x = -5, y = 0$ $\quad t = 2\pi \Rightarrow x = 0, y = 0$

$\boxed{t = 2\pi \text{ days}}$ to circle the orbit once

(b) $V_x = \frac{dx}{dt}$ $\quad V_y = \frac{dy}{dt}$ $\Rightarrow V_x = 5 \cdot \cos(t)$ $\quad V_y = 2 \cdot \cos(2t)$

$V^2 = V_x^2 + V_y^2 \Rightarrow \boxed{V = \sqrt{25\cos^2(t) + 4\cos^2(2t)}}$

(c) $t = \frac{\pi}{2} \Rightarrow x = 5 \quad y = 0 \Rightarrow (5,0,0)$

$d_1 = 5 - 4 = 1$ U.A.

$d_2 = 5 - (-4) = 9$ U.A.

$\boxed{\begin{array}{l} d_1 = 1 \text{ U.A.} \\ d_2 = 9 \text{ U.A.} \end{array}}$

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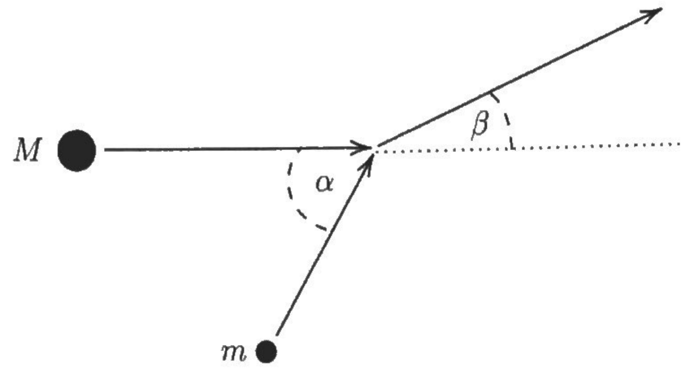
Problem B.4 : Asteroid Collision (6 Points)

A warning system has calculated that two asteroids will collide not far from Earth any time soon. The smaller asteroid has the mass m and moves with the velocity v_m. The bigger asteroid has the mass $M = 3m$ and the velocity of $v_M = \frac{1}{2}v_m$. They collide at an angle of $\alpha = 60°$ and turn into a single heavy asteroid (inelastic collision):



(a) Calculate the velocity of the single object after the collision.

(b) Determine the angle β after the collision.

(b) $Q_x = Q'_x \Rightarrow M \cdot \frac{v_m}{2} + m \cdot v_m \cdot \cos\alpha = (M+m) \cdot v \cdot \cos\beta$

$$3\cancel{m}\frac{v_m}{2} + \cancel{m} v_m \cancel{\cos\alpha}^{\frac{1}{2}} = 4\cancel{m}\, v \cos\beta \Rightarrow v = \frac{v_m}{2\cos\beta}$$

$$Q_y = Q'_y \Rightarrow m v_m \cancel{\sin\alpha}^{\frac{\sqrt{3}}{2}} = 4 m v \sin\beta \Rightarrow v = \frac{v_m\sqrt{3}}{8\sin\beta}$$

$$\frac{\cancel{v_m}}{2\cos\beta} = \frac{\cancel{v_m}\sqrt{3}}{8\sin\beta} \Rightarrow 19\sin^2\beta = 3 \Rightarrow \sin^2\beta = \frac{3}{19} \Rightarrow \sin\beta = \frac{\sqrt{57}}{19}$$

$\cos\beta = \sqrt{1-\sin^2\beta}$ $\qquad \beta = \sin^{-1}\left(\frac{\sqrt{57}}{19}\right) \Rightarrow \boxed{\beta \cong 23°24'47{,}61''}$

(a) $\cos\beta = \frac{4\sqrt{19}}{19}$

$$v = \frac{v_m}{2\cos\beta} = \frac{v_m \cdot \sqrt{19}}{8} \Rightarrow \boxed{v \cong 0.545\, v_m}$$

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Problem C.1 : The Sunburst Arc (10 Points)

This problem requires you to read following recently published scientific article:

The Sunburst Arc: Direct Lyman α escape observed in the brightest known lensed galaxy.
T.E. Rivera-Thorsen, H. Dahle, M. Gronke, M. Bayliss, J.R. Rigby, R. Simcoe, R. Bordoloi, M. Turner, and G. Furesz, Astronomy & Astrophysics 608, (2017). Link: https://iaac.space/doc/Pre-Final-Article-2017-Sunburst-Arc.pdf

Answer following questions related to this article:

(a) Why is it difficulty for *LyC radiation* to escape galaxies with high star formation rates?

Because, in this galaxies, there are a large amount of neutral hydrogen, that can be opaque to this radiation.

(b) What is the difference between the *density-bounded medium* and the *picket fence model*?

In the density-bounded medium model the neutral hydrogen isn't dense enough to be opaque to the radiation and in the picket fence model, the hydrogen doesn't completely cover the ionizing sources, so the radiation can escape.

(c) Explain the spectral shape of the *perforated shell model* (see article: figure 1, right box).

There is a high peak from the LyC and Lyα that escape through an open channel and some others peaks that are the result of the Lyα radiation that escapes from the optically thick gas.

(d) What is the *Sunburst Arc* and how was it discovered?

It is the lensed galaxy PSZ1-ARCG311.6602-18.4624 and it was discovered in an image of its lensing cluster that was discovered through its Sunyaev-Zel'dovich effect in the Planck data

(e) When did the scientist observe the object and which instruments did they use?

It was observed by Dahle et al in 2016 using the object's Sunyaev-Zel'dovich effect in the Planck data

(f) What is the redshift of the *Sunburst Arc* and how was it determined from the data?

The redshift is: $z_{neb} = 2.37094 \pm 0.00001$ and it was determined based on its FIRE spectrum by fitting a single Gaussian profile to each of the four strong emission line $H\beta$, [O III] $\lambda\lambda$(4959, 5007), and $H\alpha$, then they computed the uncertainty-weighted average of the four redshifts.

(g) Explain the difference between the right and the left diagram in figure 4 (see article).

In the left ~~right~~ diagram they fit the full observed Lyα profile to the shell model grid and in the right diagram they fit the central peak to a single Gaussian profile, which was subtracted, and the remaining profile, similar to typical double-peaked Lyα profiles, was again fitted to the shell model grid

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Problem C.2 : Dark Matter (10 Points)

This problem requires you to read following recently published scientific article:

Probing Dark Matter Using Precision Measurements of Stellar Accelerations.

A. Ravi, N. Langellier, D.F. Phillips, M. Buschmann, B.R. Safdi, R.L. Walsworth,

arXiv:1812.07578, (2018). Link: https://iaac.space/doc/Pre-Final-Article-2018-1812.07578.pdf

Answer following questions related to this article:

(a) What are the current methods to determine the dark matter density / radial velocity and what are the disadvantages of these methods?

It is determined by the Galactic rotation curve, measured via Doppler shifts, or the dispersion of local stellar velocities in the vertical direction about the Galact mid-plane, measured using astrometry. However, in these methods, is assumed equilibrium, but it is not certain.

(b) Explain the new method for measuring dark matter density that is proposed in the article.

They would use stellar accelerations, that are directly determined by the forces acting on a star, with no assumptions.

(c) Let $a_r(r)$ be the dark matter contribution to the acceleration: Show that the dark matter density is given by $\rho_{DM} \approx \frac{1}{4\pi G}\left(2(A-B)^2 - \frac{\partial a_r}{\partial r}\right)$ with Oort constants $2(A-B)^2 = 2GM(r)/r^3$.

$$\nabla \cdot a = \frac{\partial a_r}{\partial r} - \frac{2GM(r)}{r^3} \qquad \nabla \cdot a = -4\pi G \rho_{DM}$$

$$\rho_{DM} = \frac{-1}{4\pi G}\left(\frac{\partial a_r}{\partial r} - \frac{2GM(r)}{r^3}\right) \Rightarrow \boxed{\rho_{DM} = \frac{1}{4\pi G}\left(2(A-B)^2 - \frac{\partial a_r}{\partial r}\right)}$$

(d) How much does the velocity of the stars change during the lifetime of a human (80 years)?

10 year – 5 cm/s

80 years – 40 cm/s

$\boxed{\Delta V = 0{,}4\, m/s}$

$\Delta a_r = 1.5 \cdot 10^{-8}\, cm/s^2$

$\Delta a_r = 0.5\, cm/s/year$

1 year → 0.5 cm/s

(e) What is important for measuring stellar accelerations and which instruments can be used?

An extremely stable calibration of the spectrograph used to determine the RVs over several years is very important and laser frequency comb optimized for calibrating the spectrographs could be used (~~???~~ "astro-combs")

(f) Explain the curves of the four diagrams in figure 2 (see article).

In (a), its considering the acceleration due to the MW gravitational potential, in (b), stellar companions, in (c), planets (multiple planets in this case) and in (d), "noise including stellar magnetic activity and instrumental effects are depicted. So, in (a), the ~~curve~~ graphic is linear and the V_{accel} is growing, in (b) the curve is decreasing, in

(c), it is volatile over the years (it is almost a sine wave) and in (d) the average of V_{noise} is almost constant at 0.

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