International Astronomy and Astrophysics Competition - Qualification Round 2019

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LaTeX

1 Problem A : Planets and Stars

(1) 8.3 (2) solar eclipse (3) Jupiter (4) Mercury

(5) 87.97 (6) Sirius (7) 250 ± 150 (8) Andromeda

2 Problem B : The Size of Jupiter

(a).

$$\frac{V_J}{V_E} = \frac{4 \cdot \pi \cdot R_J^3}{4 \cdot \pi \cdot R_E^3} = \left(\frac{R_J}{R_E}\right)^3 \Longrightarrow \frac{V_J}{V_E} \approx 1326.4$$

Approximately 1326 Earths fit in Jupiter by volume.

(b).

$$\rho = \frac{M}{V}$$

$$\frac{M_J}{M_E} = \frac{\rho_J \cdot V_J}{\rho_E \cdot V_E} \Longrightarrow \frac{M_J}{M_E} \approx 318.97$$

Jupiter is approximately 319 times heavier than Earth.

3 Problem C : Space Race to the Moon

Alice's spaceship:

$$\Delta t_A = \frac{\Delta S}{V} \Longrightarrow t_A = \frac{3.84 \cdot 10^5}{500} \Longrightarrow t_A = 768h$$

Bob's spaceship: (Assuming $V_0 \approx 0$)

$$\Delta S = V_0 \cdot t_B + \frac{a \cdot t_B^2}{2} \Longrightarrow \Delta S = \frac{a \cdot t_B^2}{2} \Longrightarrow t_B^2 = \frac{2 \cdot \Delta S}{a} \Longrightarrow t_B = \sqrt{\frac{2 \cdot 3.84 \cdot 10^5}{1.4}} \Longrightarrow t_B \approx 740.66h$$

$$t_B < t_A$$

So even if Bob's spaceship starts with velocity 0 ($V_0 = 0$), he still wins the space race.

4 Problem D : Forces between Earth and Moon

(a). If we consider the Earth-Moon system at rest: F(r) = 0

$$\mathrm{mG} \cdot \left(\frac{M_E}{r^2} - \frac{M_M}{(d-r)^2} \right) = 0 \Longrightarrow \frac{M_E}{r^2} - \frac{M_M}{(d-r)^2} = 0$$

$$\Longrightarrow \frac{M_E}{r^2} = \frac{M_M}{(d-r)^2} \Longrightarrow M_E \cdot (d-r)^2 = M_M \cdot r^2 \Longrightarrow (d-r) \cdot \sqrt{M_E} = r \cdot \sqrt{M_M}$$

$$\Longrightarrow r \cdot (\sqrt{M_M} + \sqrt{M_E}) = d \cdot \sqrt{M_E} \Longrightarrow r = d \cdot \frac{\sqrt{M_E}}{\sqrt{M_M} + \sqrt{M_E}}$$

$$\Longrightarrow r = d \cdot \frac{M_E - \sqrt{M_E \cdot M_M}}{M_E - M_M}$$

So the distance to the Lagrange point (L_1) is:

$$r = d \cdot \frac{M_E - \sqrt{M_E \cdot M_M}}{M_E - M_M}$$

(b). When we assume that the Earth-Moon system is at rest we also assume that the spaceship is at rest and this is an approximation, since F(r) is not 0. When the spaceship is moving, it has a centripetal force, so (considering distance to the center of mass of the system : r_S and the tangencial velocity of the spaceship: V_t) :

$$F(r) = \frac{m \cdot V_t^2}{r_S}$$

5 Problem E : Polar Lights

The polar lights occur when the solar winds reach the atmosphere by the magnetic poles, since the particles that do not reach the poles are captured by the Earth's magnetic field (forming the Van Allen radiation belt). When these charged particles come in contact with the atmosphere, they excite the atoms that make up the molecules present in the atmosphere and their electrons pass to a higher energy level, but they become unstable, so when they get unexcited , the electrons come back to the original level and release photo voltaic energy, in other words: light. It can be of different colors, varying according to the element. For example, Oxygen can produce yellowish green colours ($\lambda \approx 557,7nm$) and reddish colors ($\lambda \approx 630nm$) and the Nitrogen can produce in its ionic molecular form blue-violet tones (λ between 370 and $470nm$) , while in the neutral molecular form it emits red tones (H_α spectrum).