International Astronomy and Astrophysics Competition - Qualification Round 2020

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LaTeX

1 Problem A : The Solar System

(1) Hydrogen (2) Helium (3) Astronomical Unit (A.U.) (4) Carbon Dioxide (CO_2)

(5) Asteroid Belt (6) 79 (7) Io (8) 164.8

2 Problem B : Cosmic Scales

(a)

The ratio **r** can be calculated by:

$$r = \frac{1}{1270000000}$$

Thus the Sun's diameter would be:

$$D' = D \cdot r \Longrightarrow \boxed{D' \approx 1.1\,m}$$

(b)

$$1\,ly = 2.998 \cdot 10^8 \cdot 365.25 \cdot 24 \cdot 3600 \approx 9.46 \cdot 10^{15}\,m$$

$$d' = d \cdot r = \frac{4.24 \cdot 9.46 \cdot 10^{15}}{1270000000} \Longrightarrow \boxed{3.15 \cdot 10^7\,m}$$

3 Problem C : Distance to the Moon

Rule of Three:

$$\begin{array}{ccccc} 60 & \text{cm} & — & 0.55 & \text{cm} \\ \text{x} & & — & 3500 & \text{km} \end{array}$$

$$\therefore \boxed{x \approx 3.8 \cdot 10^8\, m}$$

4 Problem D : Energy of Satellites

(a)

$$E_{kin}(h) = K = \frac{m_S \cdot v^2}{2}$$

Equilibrium of forces:

$$F_g = F_c \Longrightarrow G\frac{m_S \cdot m_E}{(R_E + h)^2} = \frac{m_S \cdot v^2}{R_E + h}$$

$$v^2 = \frac{G \cdot m_E}{R_E + h}$$

Therefore:

$$\boxed{K = \frac{G \cdot m_S \cdot m_E}{2 \cdot (R_E + h)}}$$

(b)

$$K_f = \frac{6.674 \cdot 10^{-11} \cdot 1 \cdot 5.972 \cdot 10^{24}}{2 \cdot (6.371 \cdot 10^6 + 4 \cdot 10^5)} \approx 2.94 \cdot 10^7\, J$$

For the least amout of energy needed, the satellite needs to take off on the equator:

$$K_i = \frac{m_s \cdot v'^2}{2},\ v = \frac{2 \cdot \pi \cdot R_E}{T},\ T = 24\, h = 8.64 \cdot 10^4\, s$$

$$\Longrightarrow K_i \approx 1.07 \cdot 10^5\, J$$

$$u = \frac{\Delta E}{V} \Longrightarrow V = \frac{\Delta E}{u} = \frac{2.93 \cdot 10^7}{10^6} \Longrightarrow \boxed{V \approx 29\, L}$$

5 Problem E : Nuclear Fusion

The Nuclear fusion is a process that occurs in the Sun and emits a lot of energy. Our star isn't that massive, so, it mainly does the proton–proton chain reaction, in which atoms of Hydrogen fuse and form Helium atoms, emitting other particles as well as the following equations:

$$2\,{}^{1}\mathrm{H} \to {}^{2}\mathrm{H} + \nu + \mathrm{e}^{+}$$
$$^{1}\mathrm{H} + {}^{2}\mathrm{H} \to \gamma + {}^{3}\mathrm{He}$$

$$6\,{}^{1}\mathrm{H} \to 2\gamma + 2\nu + 2\mathrm{e}^{+} + 2\,{}^{3}\mathrm{He}$$
$$2\,{}^{3}\mathrm{He} \to 2\,{}^{1}\mathrm{H} + {}^{4}\mathrm{He}$$

$$4\,{}^{1}\mathrm{H} \to 2\gamma + 2\nu + 2\mathrm{e}^{+} + {}^{4}\mathrm{He}$$

Thus, 4 Hydrogens fuse together and form one Helium, 2 gamma rays, 2 neutrinos and 2 positrons. This other particles are emited in every direction, so only a small part of each arrives in the Earth, including the photons that we see. Furthermore, we can calculate the energy that the reaction releases by the difference of masses of the particles:

$$\Delta m = 4 \cdot m_{^{1}\mathrm{H}} - m_{^{4}\mathrm{He}} \approx 4.66 \cdot 10^{-29}\, kg$$

Then, with Einstein's Equation:

$$E = \Delta m \cdot c^{2} \to E \approx 4.19 \cdot 10^{-12}\, J$$

Each reaction produces the energy **E**, therefore, since the luminosity of the sun is about $L \approx 3.83 \cdot 10^{26}\, W$, the Sun does $9.13 \cdot 10^{37}$ reactions/s (however, only photons $8.3 \cdot 10^{20}$ reaches Earth). With this information we can even estimate how much time the Sun will continue in the Main Sequence (assuming its luminosity will remain almost constant), about 10^{10} years.