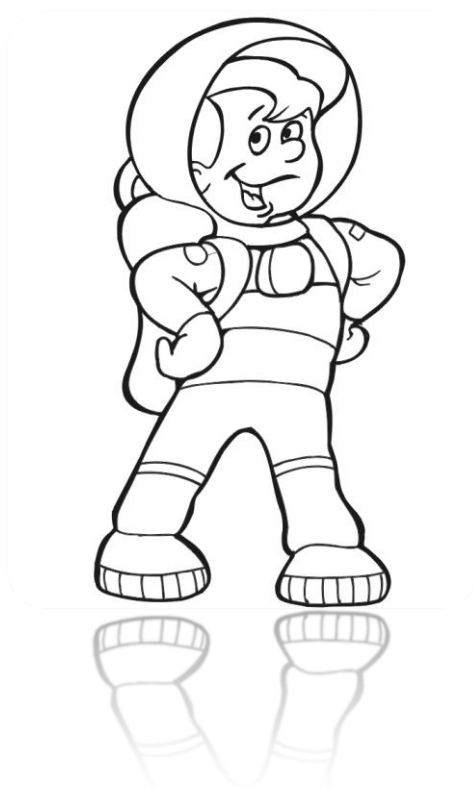


**RUSSIAN OPEN SCHOOL ASTRONOMICAL OLYMPIAD
BY CORRESPONDENCE**

(2005-2008)



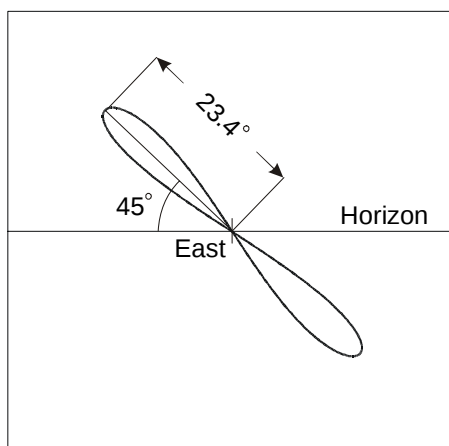
RUSSIAN OPEN SCHOOL ASTRONOMICAL OLYMPIAD BY CORRESPONDENCE – 2005

PROBLEMS WITH SOLUTIONS

1. Problem. The artificial satellite is rotating around the Earth along a circular orbit lying in the ecliptic plane. Being observed from city of Krasnodar (latitude $+45^\circ$) this satellite and the vernal equinox point always rise simultaneously. Sometimes the satellite is visible above the southern horizon. What is its altitude above the horizon in this moment? What is the radius of satellite's orbit? Refraction and daily parallax of the satellite should be ignored.

1. Solution. Anywhere on Earth (except poles) the vernal equinox point rises once in a sidereal day, about 23 hours and 56 minutes after previous rise. The moments of rise of artificial satellite are in 23 hours 56 minutes one from another. Since we ignore the parallax shift of the satellite, it is always situated at the ecliptic. At the latitude of Krasnodar ($+45^\circ$) ecliptic cannot coincide with horizon and being the major circle of celestial sphere crosses the horizon at two opposite points. One of them is rising vernal equinox point, another point is autumn equinox point. Thus, rising satellite is situated at one of these points. As the sidereal day gone, the satellite rises in the same point of the sky (being in the second point, it would not rise, but set under the horizon, since every moment just a half of the orbit is seen above the horizon).

It is obvious that the satellite rotating around the Earth cannot be immovable relatively stars. Let us consider the conditions when the satellite comes back to the same point in one sidereal day. It can take place if the rotational period is equal to one sidereal day and the rotation direction is the same with the rotation of the Earth. This orbit is like geo-stationary, but the satellite rotates not in equatorial, but in the ecliptic plane. Being observed from the Earth, the satellite draws the narrow 8-like figure with the size 47° . If it is situated at the vernal equinox point during its rise, it will rise every sidereal day at the east (see the figure).



But this picture contradicts with the conditions of the problem, since the satellite is always situated in the eastern part of the sky and is never observed above the southern horizon. We have to consider one more situation, when satellite completes the revolution around the observer on the Earth in one sidereal day, rising once and setting once in this period. Let us designate the synodic period of the satellite as S , sidereal period as T , and the duration of sidereal day as T_0 . These three values are connected by the relation

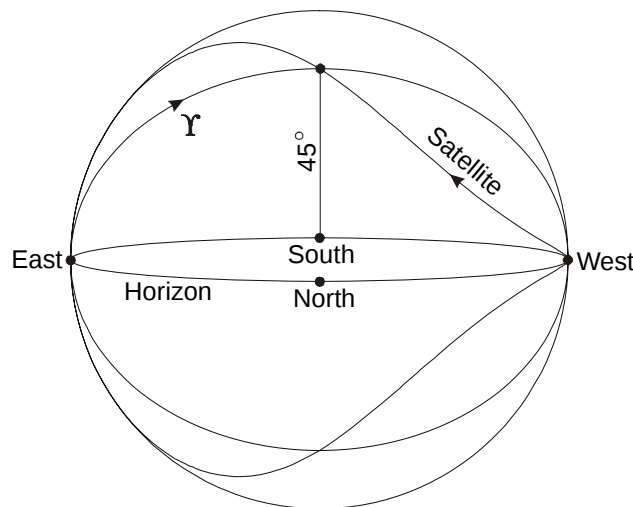
$$\pm \frac{1}{S} = \pm \frac{1}{T} - \frac{1}{T_0}.$$

The sign “+” before the values of $(1/S)$ and $(1/T)$ means the counterclockwise rotation direction (the same with the rotation of Earth), the sign “–” means the clockwise direction. Since the values of S and T_0 are

the same, the sign “–” cannot be before the value $(1/S)$, otherwise the value of T will turn to infinity. Since the value of S is positive, the solution will exist only for the following signs:

$$\frac{1}{S} = \frac{1}{T} - \frac{1}{T_0},$$

The sidereal period of the satellite T is turned out to be equal to the half of sidereal day or 11 hours and 58 minutes. The satellite rotates counterclockwise two times faster than Earth, rising at the west and setting at the east. During its rise the satellite will be situated in the *autumn* equinox, which will be setting in the same moment. Moving from the west to the east, the satellite will go towards the vernal equinox point risen at the east simultaneously with the satellite (see the second figure). In 5 hours and 59 minutes the satellite will complete a half of the revolution, come to the vernal equinox point, which will culminate at the altitude 45° above the southern horizon. We have found the answer to the first question of the problem.



To answer on the second question, we use the General Third Kepler law. The orbit radius is equal to

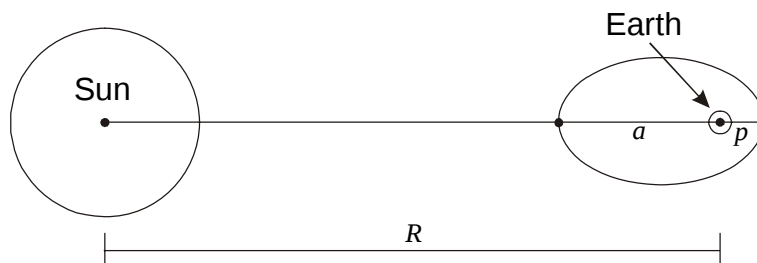
$$R = \left(\frac{GMT^2}{4\pi^2} \right)^{1/3},$$

or 26.6 thousand kilometers (here M is the mass of the Earth).

2. Problem. The artificial satellite is rotating around the Earth along an elliptical orbit lying in the ecliptic plane. When it is in perigee point, the distances from Earth to the satellite and to Moon are the same. Estimate the maximum possible eccentricity of the satellite's orbit. Please don't consider the gravitation of the Moon.

2. Solution. Possible value of orbit eccentricity is restricted by tidal influence of the Sun (the influence of Moon is disregarded). If satellite comes close to the inner Lagrange point of the system Sun-Earth, the orbit will be instable and the satellite will come to heliocentric orbit, being the artificial planet of the Solar System. Let us consider the boundary case when the apogee point of the satellite is situated between the Sun and the Earth. Since the orbit lie in the ecliptic plane, such situation takes place once in a year.

Let us designate the perigee and apogee distances of the orbit as p and a , the distance between Sun and Earth as R . Assume that the satellite in the apogee came to the inner Lagrange point. This point rotates around the Sun with the same angular velocity ω as the Earth. Let us write the motion equations for Lagrange point and the Earth:



$$\frac{GM}{(R-a)^2} - \frac{Gm}{a^2} = \omega^2(R-a);$$

$$\frac{GM}{R^2} = \omega^2 R.$$

Taking into account that $a \ll R$, we write

$$\frac{GM}{(R-a)^2} = \frac{GM}{R^2} + \frac{2GMa}{R^3}.$$

Expressing the value of ω from Earth motion equation, we receive the relation:

$$\frac{3GMa}{R^3} = \frac{Gm}{a^2}.$$

From this we make the estimation of maximal apogee distance

$$a = R \left(\frac{m}{3M} \right)^{1/3} = 1.497 \text{ mln. km}$$

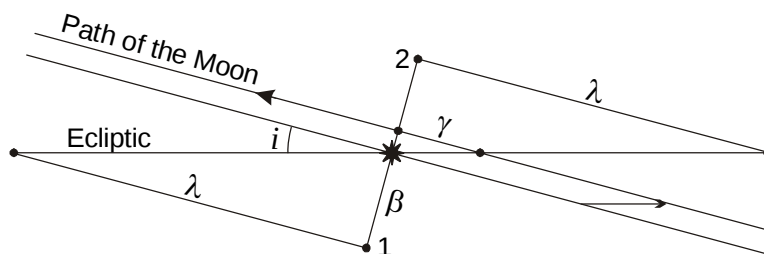
and orbit eccentricity (taking into account the value of p equal to 384 thousand kilometers):

$$e = \frac{a-p}{a+p} = 0.592.$$

3. Problem. The distant star is situated in the summer solstice point of the sky. During the passing of ascending node of lunar orbit near this star the lunar occultations of this star will be visible on the Earth every revolution of the Moon. How many occultations will this sequence contain? At what latitude and in what place of the sky will the first and the last occultation of the sequence be seen? The orbit of the Moon can be considered as circular.

3. Solution. If the Moon rotated around the Earth in the ecliptic plane, it would occult the star in the summer solstice point each revolution and this occultation would be seen in equatorial and tropical zones of our planet. But the inclination of lunar orbit to the ecliptic plane i is about 5.15° that is enough for the Moon to go above or below the star and to avoid occultation in most part of the cases. To occult the star, the Moon must be situated near one of two nodes of the orbit, where it crosses the ecliptic plane.

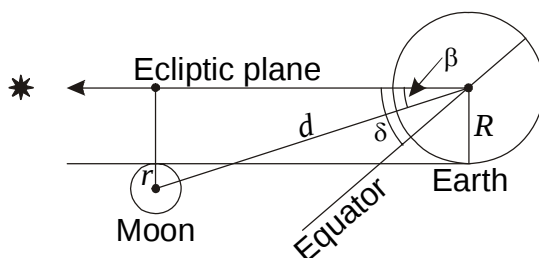
The sidereal period of lunar rotation T_S is equal to 27.321662 days and draconic period T_D (period of return to the same orbit node) is equal to 27.212220 days. Completing the revolution relatively stars, the Moon makes 1.00402 draconic revolutions. If the occultation occurs exactly in the ascending orbit node, next one will occur a month later slightly after the node crossing. The Moon will be a little bit higher and the region of occultation visibility will slightly shift northwards (see the figure).



The angular distance between the Moon and ascending node will be equal to:

$$\gamma = 360^\circ \cdot \frac{T_S - T_D}{T_D} = 1.448^\circ.$$

Shifting by this angle each sidereal month, line of nodes of lunar orbit will complete the revolution in 248.65 sidereal months or 18.6 years. This period will contain two epochs of occultation of the star near the ecliptic, near ascending and descending node, respectively. If the star is situated exactly at the ecliptic, these two sequences will be shifted by 9.3 years one from another. We have to find the duration of one sequence. Let us find the maximal distance between the star and the node for what the occultation is still possible.



Second figure shows the boundary situation, where the grazing occultation is observed just from one point on the Earth. The maximum angular distance between the Moon and ecliptic plane is equal to

$$\beta = \arcsin \frac{R+r}{d} = 1.209^\circ.$$

Here we assume the lunar orbit to be circular with radius d equal to 384.4 thousand kilometers, R and r are the radii of Earth and Moon. Returning to the first figure, we find maximal angular distance between the Moon and the orbit node:

$$\lambda = \beta \operatorname{ctg} i = 13.41^\circ.$$

The average number of occultation in one sequence is equal to:

$$N_A = \frac{2\lambda}{\gamma} = 18.52.$$

Real occultation number will be equal (with almost equal probabilities) to 18 or 19. The sequence duration will be equal to 17 or 18 sidereal months or 1.27 or 1.35 years.

In the case of ascending node the first occultation in a sequence will be near position 1 in the first figure, the same situation is shown in the second figure. The Moon will graze the star by the northern edge and this occultation will be observed in the southern hemisphere of the Earth. The Moon will be seen at the northern horizon (more exactly, at 5.15° westwards from the North point), the star declination is $+23.4^\circ$ and the occultation will be seen near Southern Polar Circle. Last occultation in a sequence (position 2 in the first figure) will be seen near North Polar Circle at 5.15° eastwards from the North point.

4. Problem. The path of total solar eclipse consistently passed the following cities: Oslo (Norway), Warsaw (Poland), Constanta (Romania), Ankara (Turkey), Baghdad (Iraq), Kerman (Iran) and Islamabad (Pakistan). What astronomical season (winter, spring, summer or autumn) the eclipse was observed in?

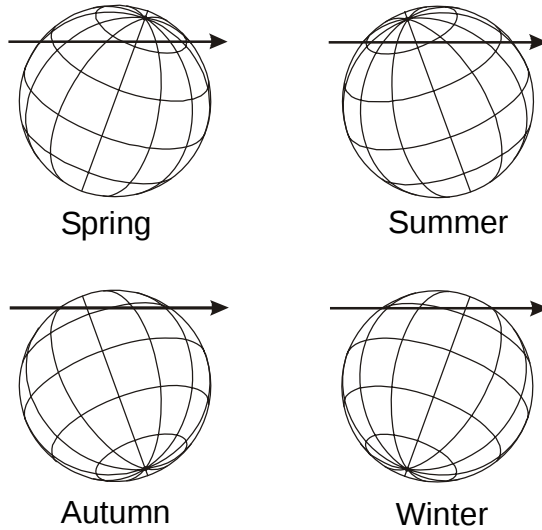
4. Solution. Table contains the coordinates of the cities listed in the problem.

City	Latitude, °	Longitude, °
Oslo	+59.9	+10.7
Warsaw	+52.2	+21.1
Constanta	+44.2	+28.6
Ankara	+39.9	+32.9
Baghdad	+33.3	+44.4
Kerman	+30.3	+57.1
Islamabad	+33.7	+73.1

As we can see, the shadow moved to the south-east for most part of time, and just in the end it moved to the east and even turned to the north-east. The shadow path is shown in the map.



Let us imagine how will the lunar shadow move on tropical and medium zone of the northern hemisphere in the middle of four seasons of the year (starting at the equinox or solstice). In the next figure we can see the Earth and shadow path at four seasons as it can be seen from the Sun (or from the Moon). The shadow moves from the left to the right (direction opposite to the motion of the Earth). Really the path motion direction is inclined to the ecliptic plane by the angle 5.15° , but it does not change the problem solution, since this angle is sufficiently less than the equator to ecliptic inclination (23.45°).

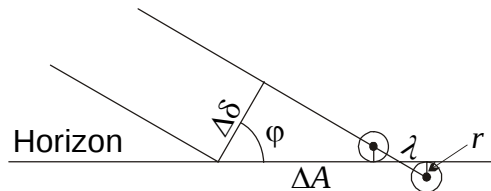


We can see that the shadow motion described in the problem can take place only during the astronomical autumn, between autumn equinox and winter solstice.

The problem has a real prototype: total solar eclipse in November, 19th, 1816, the map is above. The path of totality crossed the centers or surroundings of seven cities listed in the problem.

5. Problem. The astronomical azimuths of the rise and the following set of the planet in the city of Saint-Petersburg (latitude $+60^\circ$) were equal to -90.0° and $+90.4^\circ$. The duration of the planet disk set was equal to 3.2 seconds. What the planet is it and can it be said anything about the season, when it happened?

5. Solution. Values of azimuths show that the planet is situated near the celestial equator. Since the planets are always near the ecliptic, there were the surroundings of the vernal or autumn equinox points. The time between the rise and set of the planet is equal to 12 hours. The module of set azimuth is 0.4° higher than rise azimuth, thus the declination of the planet has increased during this time. Let us determine the value of this increase.



As we can see in the figure, change of azimuth module ΔA is related with change of declination $\Delta \delta$ and latitude of observation φ (equal to 60°) by the relation:

$$\Delta\delta = \Delta A \cos \varphi = +0.2^\circ.$$

Account of atmospheric refraction will not change this result, since the refraction changes the value of azimuth but does not change their difference (ΔA) near the celestial equator. The planet moves along the ecliptic, it is true always except the planets' stationary moments, but their angular velocity is very small that time. Near the equinox points the ecliptic creates the angle $\varepsilon = 23.4^\circ$ to the celestial parallels and equator. The total angular motion of the planet during 12 hours is equal to

$$\gamma = \frac{\Delta\delta}{\sin\varepsilon} = \Delta A \frac{\cos\varphi}{\sin\varepsilon} = 0.5^\circ.$$

Finally, the angular velocity of the planet is equal to 1 per day. It is equal to visible angular velocity of the Sun. None of the planets can reach this angular velocity by the backward motion, and only two

planets can move in the straight direction: Mercury and Venus, during their epoch of maximum elongation from the Sun.

To determine what is the planet, we determine its angular size. The duration of the planet set was equal to 3.2 seconds. Being near the equator, the planet shifted relatively observer on the Earth by the angle:

$$\lambda = 15'' \Delta t = 48''.$$

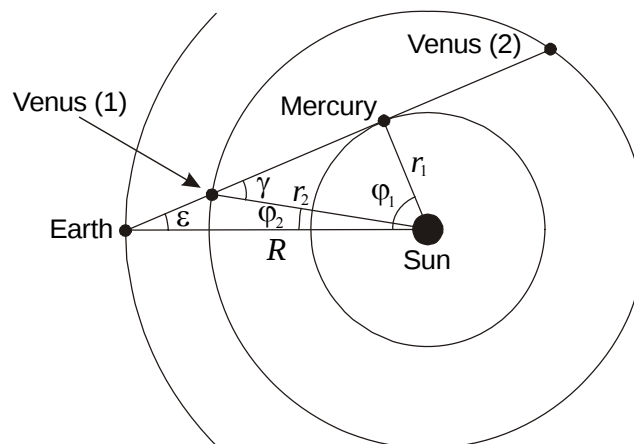
Angular diameter of the planet is equal to:

$$d = 2r = \lambda \cos \varphi = 24''.$$

This value is equal to angular diameter of Venus during the epoch of maximum elongation (the angular size of Mercury during this epoch is not larger than $8''$). Venus is at 47° from the Sun and crossing the vernal equinox point by the straight direction. If it is maximum eastern elongation, than the picture is observed in early February, if it is maximum western elongation, than it is early May.

6. Problem. Being in the maximum eastern elongation point, Mercury came to the conjunction with Venus. The angular diameter of Mercury was more than 5 times less than the angular diameter of Venus. What planet will be the first to come to the inferior conjunction with Sun? What time will it be earlier than another planet? The orbits of Mercury, Venus and Earth can be considered as circular.

6. Solution. Figure shows the positions of Mercury, Venus and Earth in the moment described in the problem.



Being in conjunction with Mercury, Venus can be in two points of the orbit signed by the digits 1 and 2 in the figure. Angular diameter of Mercury during the maximum elongation epoch is about $8''$, thus the angular diameter of Venus exceeds $40''$. It can take place only if the Venus is in the position 1, close to Earth. Let us designate the orbit radii of Mercury, Venus and Earth as r_1 , r_2 , and R , respectively, and determine the heliocentric longitude difference of Earth and each of inner planets φ_1 и φ_2 . It is easy to do for the Mercury:

$$\varphi_1 = \arccos \frac{r_1}{R} = 67.2^\circ.$$

The difference for Venus can be calculated from the triangle Sun-Venus-Earth with account of adjacent angle properties:

$$\varphi_2 = \gamma - \varepsilon = \arcsin \frac{r_1}{r_2} - \arcsin \frac{r_1}{R} = 9.6^\circ.$$

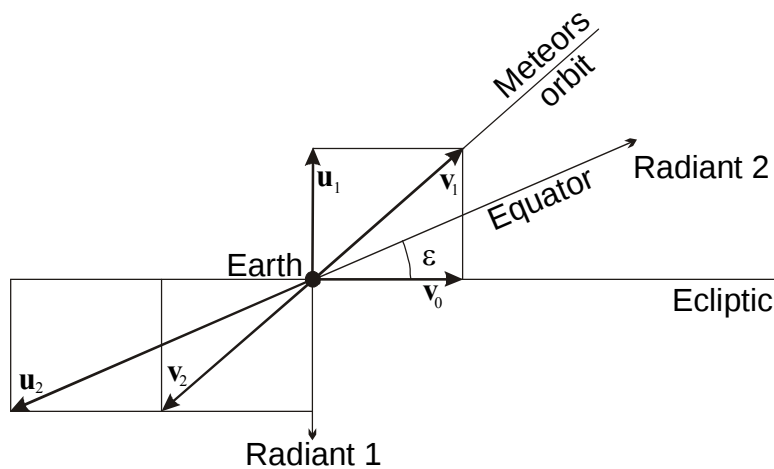
The time until the inferior conjunction of inner planet is equal to:

$$T_{1,2} = \frac{\varphi_{1,2}}{360^\circ} \cdot S_{1,2},$$

where S is the synodic period of inner planet, that is equal to 115.9 days for Mercury and 583.9 days for Venus. Substituting the numerical data, we obtain that inferior conjunction of Mercury will occur in 21.6 days and inferior conjunction of Venus will occur in 15.6 days. Finally, Venus will come to inferior conjunction in 6 days earlier than Mercury.

7. Problem. Two meteor streams are moving around the Sun by the same orbit, but in opposite directions. One moment both streams meet each other and the Earth. In the same moment two meteor showers are visible on the Earth, their radiant points have the coordinates $\alpha = 6^h$, $\delta = -66.6^\circ$ and $\alpha = 18^h$, $\delta = 0^\circ$. Please find the eccentricity of the meteors' orbit. What date did the streams meet the Earth? The orbit of Earth can be considered a circular one.

7. Solution. Since two meteor streams move by the same orbit in different directions, they will have equal velocities with opposite directions in the moment of Earth flyby. The radiant shows the directions opposite to geocentric velocity, that's why the radiants of two showers are not situated in opposite points of the sky.



Let us designate the vector of Earth's velocity as $\mathbf{v}_0$, the vectors of heliocentric velocities of meteor streams as $\mathbf{v}_1$ and $\mathbf{v}_2$, and their geocentric velocities as $\mathbf{u}_1$ and $\mathbf{u}_2$. For these velocities we can write the relations:

$$\begin{aligned}\mathbf{u}_1 &= \mathbf{v}_1 - \mathbf{v}_0, \\ \mathbf{u}_2 &= \mathbf{v}_2 - \mathbf{v}_0 = -\mathbf{v}_1 - \mathbf{v}_0.\end{aligned}$$

Adding them to each other,

$$\mathbf{u}_1 + \mathbf{u}_2 = -2\mathbf{v}_0.$$

The radiant of first shower is situated in the South Ecliptic pole, thus, the figure plane containing the vector $\mathbf{u}_1$ is perpendicular to the ecliptic plane crossing it by the line containing the vector $\mathbf{v}_0$. Both vectors are perpendicular to the radius-vector of Earth, directed from the center of the Sun, and the whole figure plane including heliocentric meteor velocities is perpendicular to the line Sun-Earth. Thus, during the flyby the meteors were in the perihelion or aphelion of their orbit if this orbit is not circular.

During the event Earth is moving towards the ecliptic point that is one of two cross points of ecliptic and major circle of picture plane. The last one is the declination circle containing all sky points with right ascension values equal to 6^h and 18^h (it is easy to see basing on the radiants coordinates). Thus, these two points are the solstice points. As we can see in the last equation, the projections of the vectors $\mathbf{u}_2$ and $\mathbf{v}_0$ on the ecliptic have different signs (since the vector $\mathbf{u}_1$ is perpendicular to the ecliptic). Taking into account the coordinates of the second radiant we resume that the motion of Earth is directed to the winter solstice point. Such direction takes place during the vernal equinox, March, 21st. It is the answer to the second question of the problem.

The angle between the directions to the second radiant (celestial equator) and Earth velocity direction ε is equal to 23.4° . Projecting the vector equation for the sum of velocities $\mathbf{u}_1$ и $\mathbf{u}_2$ on two coordinate axes, we obtain:

$$\begin{aligned} u_1 &= u_2 \sin \varepsilon, \\ 2v_0 &= u_2 \cos \varepsilon \end{aligned}$$

and then

$$u_1 = 2v_0 \operatorname{tg} \varepsilon.$$

Heliocentric meteor velocity during the meeting event is equal to

$$v_1 = \sqrt{u_1^2 + v_0^2} = v_0 \sqrt{1 + 4 \operatorname{tg}^2 \varepsilon} = 39.4 \text{ km/s}.$$

This velocity exceeds the circular velocity v_0 but is less than parabolic velocity. Thus, the orbit of the meteors is elliptical and they are crossing the perihelion. Denoting the distance between Sun and Earth as r , we express the perihelion velocity:

$$v_1 = \sqrt{\frac{GM}{a} \cdot \frac{1+e}{1-e}} = \sqrt{\frac{GM}{r} (1+e)} = v_0 \sqrt{1+e}.$$

Here a is the large semi-axis of the meteor orbit, e is its eccentricity and M is the solar mass. Finally,

$$e = 4 \operatorname{tg}^2 \varepsilon = 0.75.$$

8. Problem. While using the Giant Optical Cosmic telescope with ultra-high resolution astronomers of the future succeeded to see the disk of Betelgeuse (α Orionis). What object – Betelgeuse or Venus – will have higher surface brightness (the brightness of angular square unit)? What will the ratio of surface brightness of these objects be?

8. Решение. To solve the problem, we must remember that in the absorption-free case the surface brightness of the object does not depend on the distance. The total brightness is back proportional to the square of distance, but the same is the visible square of the object. For the self-emitting stars in the black body assumption the surface brightness is the function of surface temperature, increasing proportionally to its fourth degree. Betelgeuse is the red supergiant of spectral type M2 with the surface temperature about 3000 K, twice less than the solar one. Thus, surface brightness of Betelgeuse is about 16 times less than the one of the Sun.

Venus does not emit in the visible part of spectrum, but reflects the solar radiation. The ratio of surface brightness of Venus (center of the disk near superior conjunction) and the Sun is equal to

$$\frac{j_V}{j_0} = A \frac{r^2}{R^2} = \frac{1}{37300}.$$

Here A is the albedo of Venus, r is the radius of Sun and R is the orbit radius of Venus. Finally, surface brightness of Betelgeuse is about 2330 times higher than the one of Venus.

9. Problem. The globular stellar cluster has the magnitude equal to 4.5^m and angular diameter equal to $25'$. The distance to the cluster is equal to 3 kpc. Having assumed that the cluster consists of the stars, which are like the Sun and their density is the same over the whole ball volume, so estimate the brightness of the night sky of the planet rotating around the star in the center of cluster. Compare it with the moonlit sky on the Earth. The absorption of light in the interstellar medium and planet's atmosphere can be ignored.

9. Solution. Expressing the angular diameter of stellar cluster in radians multiplying it to the distance and dividing by 2, we obtain the cluster radius r_0 equal to 10.9 pc. The cluster volume is equal to

$$V = \frac{4}{3} \pi r_0^3 = 5.42 \cdot 10^3 \text{ pc}^3.$$

Absolute magnitude of the Sun M_0 is equal to $+4.7^m$. At the distance d equal to 3 kpc, Sun would be seen as the star with magnitude

$$m_0 = M_0 - 5 + 5 \lg r = 17.1.$$

Denoting the total cluster magnitude as m , we express the number of stars in the cluster:

$$N = 10^{0.4(m_0 - m)} = 1.095 \cdot 10^5$$

and the stars volume density:

$$n = \frac{N}{V} = 20.2 \text{ pc}^{-3}$$

In the night sky of the planet inside the cluster the stars of a half of the ball is seen, and they fill the sky uniformly. Let us designate the illumination from one star with magnitude 0^m as J and calculate the illumination from all stars in a thin semi-spherical shell with radius r and thickness Δr . The number of the stars in a shell is equal to

$$N_r = 2\pi r^2 \Delta r n.$$

The magnitude and illumination from each star are equal to

$$m = M_0 - 5 + 5 \lg r,$$

$$j_r = \frac{J \cdot 10^{2-0.4M_0}}{r^2}.$$

Here r is designated in parsecs. The illumination from all stars in a shell is equal to

$$J_r = 2\pi n J \cdot 10^{2-0.4M_0} \cdot \Delta r.$$

We see that this value is proportional to the thickness of the shell and does not depend on its radius. Performing the whole semi-sphere as the composition of such shells, we obtain the expression of the total illumination of planet night sky:

$$J_T = 2\pi n J \cdot 10^{2-0.4M_0} \cdot r_0 = J \cdot 1.83 \cdot 10^3.$$

The total magnitude is equal to:

$$m_T = -2.5 \lg \frac{J_T}{J} = -8.2.$$

The sky is sufficiently brighter than the moonless sky on the Earth, and the objects outside the clusters will make just a small contribution to this value. But the moonlit night on the Earth (the Moon magnitude is equal to -12.7^m) is 63 times brighter than the sky of the planet in the center of the globular cluster.

10. Problem. The radius of the Galaxy is equal to 15 kpc, the thickness of its disk being many times less. The mass of the galaxy is equal to 10^{11} solar masses and it is distributed uniformly in the volume of the galaxy. Two stars are rotating around the center of the galaxy in the same direction by the circular orbits with radii equal to 5 kpc and 10 kpc. Please find the synodic period of the first star while observing from the vicinity of the second star.

10. Solution. The mass density inside the galaxy is equal to

$$\rho = \frac{M}{\pi R^2 d}.$$

Here M , R and d are the mass, radius and thickness of the galaxy. We assume that the motion of star by the circular orbit with radius r is influence only by the part of the galaxy inside the cylinder with the same radius. Mass of this part is equal to

$$m(r) = \pi r^2 d \rho = M \frac{r^2}{R^2}.$$

The angular velocity of the star's rotation around the center of the galaxy is equal to

$$\omega(r) = \sqrt{\frac{Gm(r)}{r^3}} = \sqrt{\frac{GM}{R^3} \cdot \frac{R}{r}} = \omega_0 \sqrt{\frac{R}{r}}.$$

Here ω_0 is the angular velocity of the rotation of galaxy edge. Note that such dependency is weaker than the one by Kepler law (with the degree $-3/2$). Synodic period of the star with orbit radius r_1 observed from the star with radius r_2 is equal to

$$S = \frac{2\pi}{\omega(r_1) - \omega(r_2)} = 2\pi \sqrt{\frac{R^3}{GM}} \cdot \frac{1}{\sqrt{R/r_1} - \sqrt{R/r_2}}$$

Substituting the numeric values, we obtain 1.07 billions of years.

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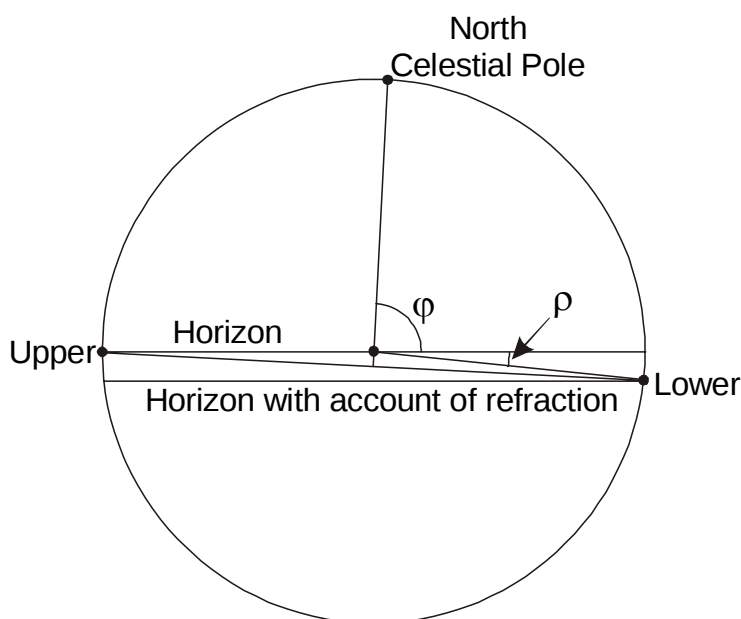
PROBLEMS WITH SOLUTIONS

1. Problem. Due to atmosphere refraction ($34'$ near the horizon) the object that would not rise in definite location on the Earth, does not set under the horizon, always being above. At what latitudes on the Earth this can happen?

1. Solution. This situation can take place if the whole daily path of the celestial object is situated not deep below the horizon (the altitude is higher than $-34'$). This can be in two cases: if the daily path has small angular size or if it is close to parallel to the horizon.

Small angular size means that the object is situated near North or South Celestial Pole. If the path (the circle with small radius) is near to horizon, then the observations must be carried out near the equator. The equator (exactly) does not meet the problem conditions, since there are no invisible objects there even in the case of absence of refraction. But having made a small step from there, for example, northwards, the South Celestial Pole will drop down below the horizon, but until the latitude $+0^\circ 34'$ it will be visible owing to refraction. Finally, in the first case the problem statement is true in two bands around equator, up to latitudes $0^\circ 34'$ (north and south).

The second case can be observed near North and South Poles of the Earth. If we come to the North Pole, the problem statement will be true, since the objects with declinations from 0° to $-0^\circ 34'$ will be always above the horizon due to refraction. However, the statement will remain true at some distances from the pole. Let's consider the boundary situation: the object is at altitude 0° in upper culmination and $-0^\circ 34'$ in lower one (see the figure in the projection to the celestial meridian plane):



The points of upper and lower culminations are situated at equal distances from the North Celestial Pole. Let φ will be the latitude, ρ is the refraction. This case we will have:

$$180^\circ - \varphi = \varphi + \rho,$$

From this equation we obtain the value of latitude φ :

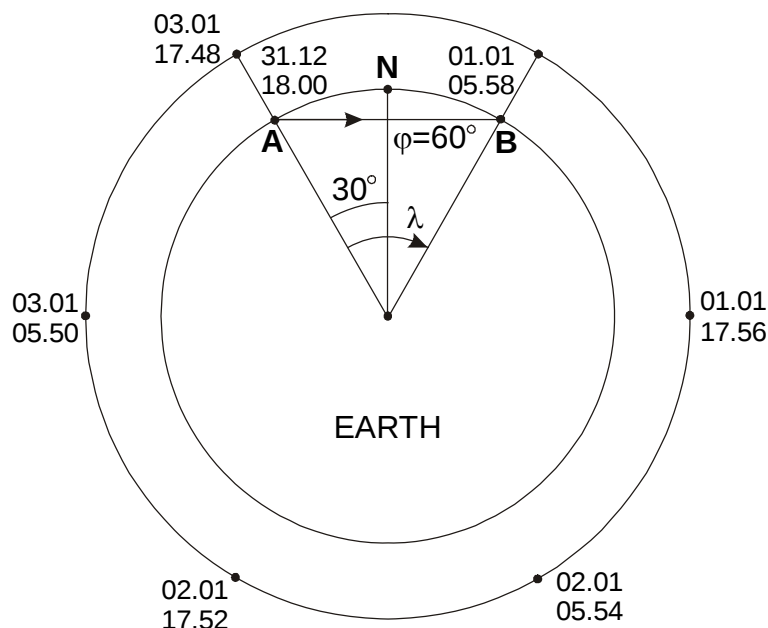
$$\varphi = 90^\circ - (\rho/2) = 89^\circ 43'.$$

The declination of the object will be equal to $-0^{\circ}17'$. Having made further step from the pole, the altitude difference in the culminations will be higher than $34'$, and the problem statement will not be true. The analogous calculations can be done for surroundings of Southern Pole of the Earth.

The final answer is following: this situation can be observed at the latitudes: $[-90^{\circ}, -89^{\circ}43']$, $(-0^{\circ}34', 0^{\circ})$, $(0^{\circ}, +0^{\circ}34')$, $(+89^{\circ}43', +90^{\circ})$.

2. Problem. Amateur astronomers observe the artificial satellite of the Earth in Saint-Petersburg. The satellite crossed the zenith twice during the New Year night – at 18.00, December, 31st and at 05.58, January, 1st. When will the satellite reach the zenith again? The latitude of Saint-Petersburg is equal to $+60^{\circ}$, the satellite orbit is circular.

2. Solution. The period between two moments of zenith crossing in Saint-Petersburg is equal to 5h 58m or a half of sidereal day. The plane of satellite orbit contains the center of the Earth and positions of Saint-Petersburg in these two moments (points **A** and **B** in the figure). This plane goes through the poles of the Earth.



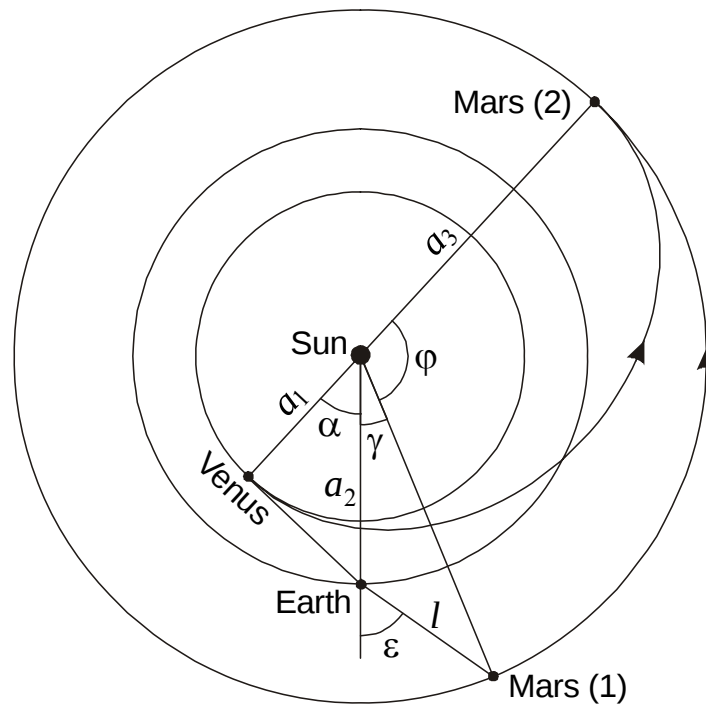
The parallel $+60^{\circ}$ containing Saint-Petersburg (its motion shown by arrow in the figure) crosses the plane of satellite orbit in two points **A** and **B** and satellite can be observed in the zenith only when Saint-Petersburg is situated in one of these two points. The orbit of satellite will cross the zenith in Saint-Petersburg twice a sidereal day, or once in 11 hours and 58 minutes.

At 5h 58m, January, 1st, satellite displaced by the angle λ , equal to 60° , relatively its position at 18h, December, 31st. It is not necessary that satellite had made $1/6$ part of its revolution around the Earth, moved above the North Pole. It can make $5/6$ revolutions, moved above South Pole, or $(n+1/6)$ or $(n+5/6)$ revolutions, where n – positive integer number. But in any case its displacement for $1/2$ sidereal day is equal to 60° . In another half of sidereal day, at 17h 56m, January, 1st, the Saint-Petersburg will be in the point **A** again, but the angle λ will be equal to 120° and satellite will come to the equator plane. It will not be seen in the zenith in Saint-Petersburg, actually it will be not seen from there.

During next two crosses of orbit plane by Saint-Petersburg (05h 54m and 17h 52m, January, 2nd, points **B** and **A**, respectively), the satellite will be situated in the southern hemisphere, and at 05h 50m, January, 3^d (Saint-Petersburg in the point **B**), it will be at equator again. Finally, at 17h 48m, January, 3^d, the satellite will be in the same point of the orbit as at 18h 00m, December, 31st, and Saint-Petersburg will be in the point **A**. The satellite will reach the zenith again.

3. Problem. In the New Year evening Venus reaches the maximum eastern elongation point being observed from the Earth. In the same time the spacecraft is launched from the Venus to the Mars by the orbit tangent to the orbits of Venus and Mars. What bright star is seen near to Mars in this New Year evening in the Earth's sky?

3. Solution. Figure shows the positions of Venus and Earth in the New Year Evening, when the spacecraft was launched from Venus.



Let's designate the radii of the orbits of Venus, Earth and Mars as a_1 , a_2 and a_3 , respectively. Their values are 0.723, 1 and 1.524 a.u. Since Venus is seen from Earth at maximal eastern elongation, line connecting Venus and Earth will be tangential to the Venus orbit. The heliocentric angle α between the directions to the Venus and Earth will be equal to

$$\alpha = \arccos \frac{a_1}{a_2} = 43.7^\circ.$$

The point on the orbit of Mars, where the craft will reach the planet, is situated in the direction opposite to the current direction to the Venus. To find the current position of Mars, we will calculate the duration of the craft flight. The major semi-axis of its orbit is equal to

$$d = \frac{a_1 + a_3}{2}.$$

The flight duration is the half of the orbital period. Expressing it in years, we obtain from Third Kepler law:

$$T = \frac{1}{2} \left(\frac{a_1 + a_3}{2} \right)^{3/2}.$$

During this period Mars will displace by the angle

$$\varphi = 360^\circ \cdot \frac{\frac{1}{2} \left(\frac{a_1 + a_3}{2} \right)^{3/2}}{a_3^{3/2}} = 113.9^\circ.$$

In the launch moment Mars was moving by the orbit ahead of Earth by the angle γ :

$$\gamma = 180^\circ - \alpha - \varphi = 22.4^\circ.$$

The distance between Earth and Mars was equal to

$$l = (a_2^2 + a_3^2 - 2a_2a_3 \cos \gamma)^{1/2} = 0.710 \text{ a.u.}$$

The geocentric angle ε between directions to Mars and anti-solar point can be found from sine theorem:

$$\varepsilon = \arcsin\left(\frac{a_3}{l} \sin \gamma\right) = 54.9^\circ.$$

Finally, in the New Year night Mars was situated about 55° eastwards from anti-solar point, which was in the western part of Gemini constellation. Thus, Mars was in the west part of Leo constellation, near its brightest star, Regulus.

4. Problem. Which stars from the list following will be visible in Moscow (latitude $+56^\circ$) in 13 000 years: Sirius, Canopus, Vega, Capella, Arcturus, Rigel, Procyon, Altair, Spica, and Antares.

4. Solution. 13 000 years is the half of the Earth axis precession period. During this time the North Celestial pole will make the half of the revolution around North Ecliptic pole ($\alpha=18\text{h}$, $\delta=+66.6^\circ$). The North Celestial pole will come to Hercules constellation to the point which coordinates now are equal to $\alpha=18\text{h}$, $\delta=+43.2^\circ$.

Since the Earth axis rotated around North Ecliptic pole, the sky areas situated near ecliptic now will always remain near the ecliptic and will be visible above the horizon in Moscow. Thus, Spica and Antares will be seen in 13000 years, note that they be seen better than now.

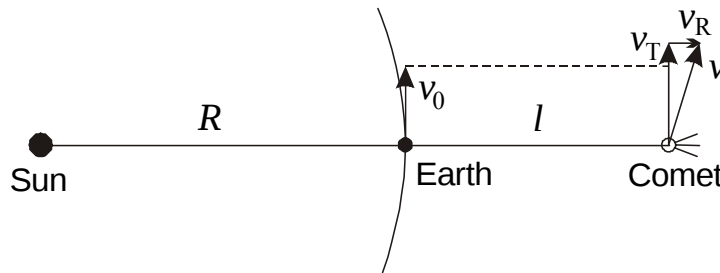
The stars situated in the Northern Ecliptic hemisphere, will remain there again being visible from Moscow. These are the stars Vega, Capella, Arcturus and Altair. Vega will be close to North Celestial pole.

Four stars remained will not be visible in Moscow. Canopus is close to South Ecliptic pole, always remaining there. Sirius, Rigel and Procyon have the right ascension close to 6 hours now. In 13000 years the North Celestial pole will move by 47° from them, their declination will decrease by about 47° . Procyon will be visible only in the souther regions of Russia, Sirius and Rigel will be invisible even there.

The account of proper motion of the stars does not change the answer. From ten stars listed in the problem just three have proper motion higher $1''$ per year: Arcturus ($2.3''$ per year), Sirius and Procyon ($1.3''$ per year for both stars). Even these nearby stars will displace only by 8.3° and 4.7° respectively, note that Sirius and Procyon move southwards, which will made the conditions of observations from northern latitudes even worse.

5. Problem. Bright comet comes to the opposition with the Sun, moving in the sky along the ecliptic in the straight direction (from the west to the east). Estimate the maximum possible distance between the Earth and the comet at this moment.

5. Solution. Since the comet is in opposition with the Sun, it is further from Sun than Earth. Since the comet moves along the ecliptic line in the sky, it moves in the ecliptic plane in the space.



Being observed from Earth, the comet moves (relatively stars) from the west to the east, to the same direction as Earth rotation around the Sun. This can happen if tangential (relatively Sun) velocity of the comet v_T is higher than orbital velocity of the Earth v_0 . Independently on the value of comet radial velocity v_R , its full velocity exceeds the velocity of the Earth.

The comets orbits are usually high-eccentricity ellipses or parabolas. The orbit can be even hyperbolic, but very close to parabolic (eccentricity is not higher than 1.001). Thus we can assume that the spatial velocity of the comet can't exceed the parabolic velocity for current distance from the Sun. Taking into account that the Earth's orbit is close to circular, we obtain:

$$v_0 = \sqrt{\frac{GM}{R}} < v \leq \sqrt{\frac{2GM}{R+l}}.$$

Here M is the solar mass, R is the radius of the Earth's orbit and l – the distance between Earth and comet during the opposition. Finally we find that the distance to be calculated, l , is less than R , or less than 1 astronomical unit.

6. Problem. The white dwarf with radius 6000 km, surface temperature 10000 K and mass equal to solar one moves through the interstellar cluster of comet cores, each one has radius 1 km and density 1 g/cm³. How many comets must fall on the white dwarf every day to increase its luminosity in two times?

6. Solution. For the beginning, we have to calculate the white dwarf luminosity:

$$L = 4\pi\sigma R^2 T^4 = 2.56 \cdot 10^{23} \text{ Watts}$$

or $6.6 \cdot 10^{-4}$ of solar luminosity. Here R and T – radius and surface temperature of white dwarf, σ – Stefan-Boltzmann constant. During 1 day (86400 seconds) white dwarf emits the energy E_0 , equal to $2.21 \cdot 10^{28}$ Jouls.

The mass of comet core is equal to

$$m = \frac{4}{3}\pi\rho r^3 = 4.19 \cdot 10^{12} \text{ kg}.$$

Being falling on the white dwarf, the core releases the energy

$$E = \frac{GMm}{R} = 9.2 \cdot 10^{25} \text{ Jouls}.$$

Here M is the white dwarf mass. To provide the amount of energy equal to luminosity of white dwarf, $(E_0/E) \sim 240$ comets must fall on the white dwarf daily. It is a good estimation, white dwarf luminosity

will be influenced by two opposite effects. From the one side, not the whole amount of falling core energy will transfer to visible emission, and from the other side, accretion of substance to the white dwarf will lead to its further compression and energy emission. If the mass value reach 1.4 solar mass, the white dwarf will explode as I type Supernova.

7. Problem. What should be the size of hypothetical molecular hydrogen cloud with density equal to the one of near-ground air and temperature 1000 K to create the star?

7. Solution. The hydrogen in the nature is performed basically by its isotope ^1H . Since the electron mass is very small compared with proton one, the mass m of hydrogen molecule H_2 is close to the double proton mass: $3.3 \cdot 10^{-27}$ kg.

Consider the ball-type cloud of molecular hydrogen with radius R , temperature T and mass density ρ . To be stable and not to be scattered in the space, the cloud must have the thermal particle velocity less than parabolic velocity on the border of the cloud:

$$\frac{3kT}{m} < \frac{2GM}{R} = \frac{8\pi G\rho R^2}{3}.$$

Here k – the Boltzmann constant, M – mass of the cloud. This leads to the expression of cloud radius:

$$R > \sqrt{\frac{9kT}{8\pi Gm\rho}}.$$

For the temperature 1000 K and mass density 1.23 kg/m^3 we obtain the value of minimal cloud radius: 135000 km.

This seems to be the answer of the problem. But if we calculate the cloud mass with minimal radius, we will obtain $1.27 \cdot 10^{25}$ kg or just 2 masses of the Earth. Having this radius and mass, the cloud will collapse creating the planet, but not the star. More exact calculations using Jeans criteria give the values of minimal radius (260000 km) and mass ($9 \cdot 10^{25}$ kg), which is again not enough to create the star.

To answer on the question, we have to calculate the cloud radius, when mass reaches the value M_* – minimum stellar mass, 0.08 solar mass or $1.6 \cdot 10^{29}$ kg. This mass is necessary for hydrogen cycle nuclear reactions to start. The minimal radius of the cloud will be

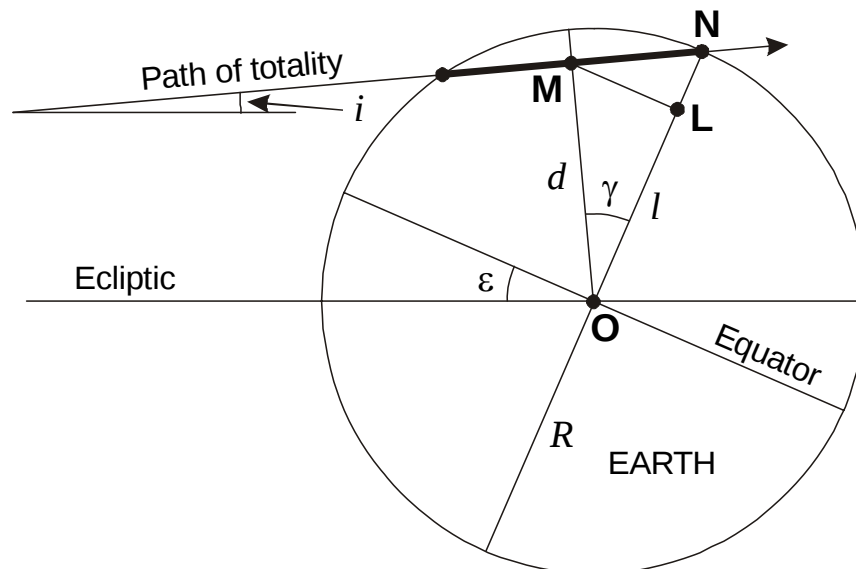
$$R_* = \left(\frac{3M_*}{4\pi\rho} \right)^{1/3} = 3140\,000 \text{ km}.$$

It is obvious that such a cloud will not scatter in the space and will collapse.

The problems about total solar eclipses

8. (The problem about springtime eclipse). Problem. The total solar eclipse occurs at the spring equinox day. The path of totality crosses the North Pole of the Earth. At what latitude on the Earth will the central eclipse be observed at the maximum altitude over horizon? Please find the value of this altitude. During the eclipse the Moon is situated near ascending node of the orbit.

8. Solution. Let's look to the Earth and path of totality on its surface from the Sun (and the Moon).



At the day of spring equinox the equator will be seen in this projection as the line inclined to the ecliptic by the angle ε equal to 23.4° . The system "Earth-Moon" moves as a single whole along the ecliptic, but this motion does not matter for this problem. The basic is the motion of the Moon and its shadow relatively the center of the Earth, which is shown as arrow in the figure. It is directed to the right, since the Moon rotates counterclockwise around the Earth, if we look from the north. The Moon is situated near ascending node of the orbit and moves (with the shadow) not parallel to the ecliptic plane, but by the angle i , equal to 5.2° .

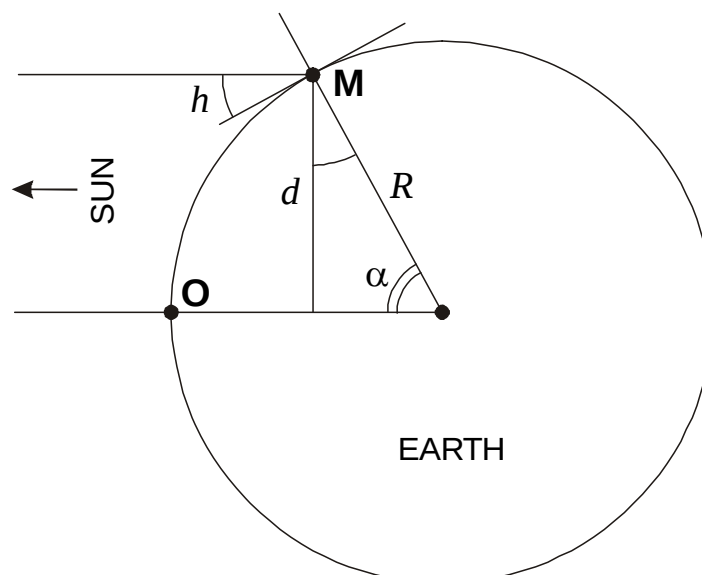
Path of totality goes through North Pole of the Earth (point N in the figure). The angle γ is equal to the inclination of totality path to the equator:

$$\gamma = \varepsilon + i = 28.6^\circ.$$

Central eclipse at maximal altitude will be visible at the point M , closest to the point O , where Sun is visible in the zenith. The length of segment MO (in this projection) is equal to

$$d = R \cos \gamma,$$

where R is the radius of the Earth. To determine the Sun altitude in the point M , we will look to the picture in side projection relatively the direction Sun-Moon (next figure):



From the picture geometry, the Sun altitude h is equal to:

$$h = \arccos \frac{d}{R} = \gamma = 28.6^\circ.$$

Let's find the latitude of the point M. Let's come back to the first figure and draw the line parallel to the equator from the point M. It will be the projection of the Earth's parallel, or the line of equal latitudes. This line will cross the meridian projection **ON** in the point **L**. The length of the segment **OL** will be equal to

$$l = d \cos \gamma = R \cos^2 \gamma.$$

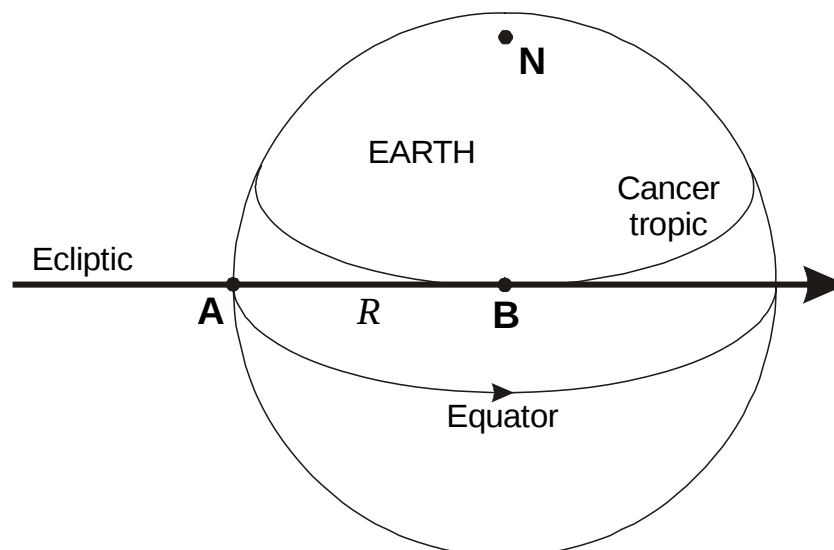
Now we can calculate the latitude by the way analogous to the altitude calculations, the latitude is analogous to the angle α in the second figure. Its value is equal to

$$\varphi = \arcsin \frac{l}{R} = \arcsin (\cos^2 \gamma) = 50.4^\circ.$$

It is important to note that the maximum eclipse altitude (28.6°) is not equal to the Sun upper culmination altitude at the latitude 50.4° . It is explained by the fact that the greatest eclipse will be observed not at the noon, but during the first half of the day. Rotation of the Earth influences only on the longitude of the point **M**, does not changing the answer of the problem.

9. (The problem about summertime eclipse). Problem. The total solar eclipse occurs at the summer solstice day. The shadow of the Moon enters the Earth surface at the point with latitude and longitude both equal to 0° . The totality duration at this point is equal to 1 minute. Please find the maximum totality duration for the stationary observer on the Earth, the coordinates of the point when it will be observed and the Universal Time of the middle of the longest total eclipse. The inclination of the lunar orbit, the refraction and the time equation can be neglected.

9. Solution. As in the Problem 8, we look to the Earth from the Sun (or Moon):

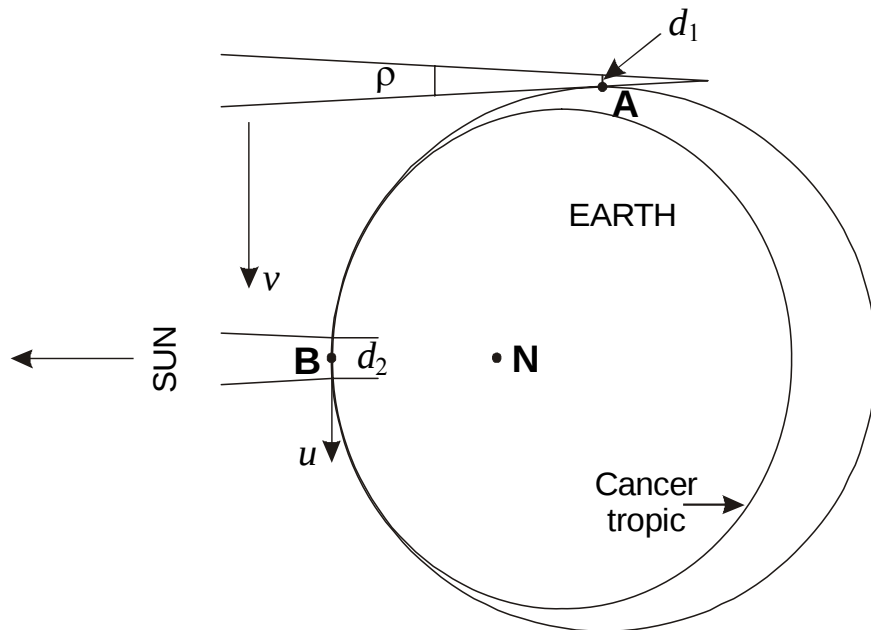


Since the eclipse occurs in the summer solstice, the equator will be seen as the semi-ellipse, grazing the limb in two opposite points lying on the ecliptic line. The path of totality enters the Earth in the one of these points, point **A**. Since we ignore the inclination of the lunar orbit to the ecliptic plane, the path will coincide with the ecliptic line, crossing the Earth by diameter. The greatest eclipse will occur in the point **B**, the center of the Earth disk if we look from the Sun. The totality maximum will be

observed at the zenith and thus the latitude of this point is equal to the declination of the Sun ($+23^\circ 26'$), and this point is lying on the Cancer tropic.

Let's look on the Earth from the North Ecliptic pole (the second figure). The Sun is situated many times farther than the Moon, and the shadow is the cone with the top angle equal to angular diameter of the Sun ρ ($31.5'$ at the summer solstice day). This cone moves relatively Earth with the same velocity v as Moon does (1.02 km/s). The width of the cone near the point **A** is equal to

$$d_1 = vT_1 = 61.2 \text{ km.}$$



Here T_1 is the totality duration in the point **A**. When the shadow reaches point **B**, this duration increases by two reasons. First, this point is closer to the Moon, and the shadow diameter will be equal to

$$d_2 = d_1 + \rho R = 119.6 \text{ km.}$$

Here R is the Earth radius, Sun angular diameter ρ is expressed in radians. Second, observed in the point **B** moves with the rotating Earth with the velocity

$$u = \frac{2\pi R \cos \varphi}{T_0} = 0.43 \text{ km/c,}$$

and this velocity has the same direction with the shadow (here T_0 is the solar day duration). Finally, the totality duration reaches the value

$$T_2 = \frac{d_2}{v - u} = 203 \text{ c}$$

or 3 minutes and 23 seconds. The values remained to find is the longitude of point **B** and Universal Time of greatest eclipse.

When the shadow enters the Earth in point **A** ($0^\circ, 0^\circ$), there was the sunrise. Since we disregard the refraction and time equation, the sunrise was at 6^{h} in the morning or 6^{h} UT. To reach point **B**, the shadow needs the time

$$\tau = \frac{R + (d_1 / 2)}{v},$$

or 1 hour and 45 minutes. Thus, the greatest eclipse was observed at 7^h45^m UT at the upper culmination of the Sun at the zenith. Disregarding the time equation, the longitude of point **B** is equal to 4^h15^m or +63°45'. The latitude, as shown above, is equal to +23°26'.

10. (The problem about solar corona). Problem. It is known that the free electrons scatter the emission almost isotropically as the metal balls with radius equal to $4.6 \cdot 10^{-15}$ meters, but heavy particles (protons, ions, atoms) scatter the light many times worse. Assuming that the corona consists of pure hydrogen, the atmospheric pressure in the lower corona layers is equal to 0.003 Pascals and the average corona temperature is equal to 1 000 000 K, estimate the magnitude of the Sun during the total solar eclipse on the Earth.

10. Solution. The corona is quite hot. At such temperature the corona hydrogen will be totally ionized and all electrons will be free. The basic contribution to the brightness is made by inner corona regions. Disregarding the changes of gravity acceleration g with the altitude in these regions, we express the atmospheric pressure in the lower border of the corona:

$$p = \mu g = \frac{GM\mu}{R^2}.$$

Here M and R are mass and radius of the Sun, μ is the column mass of corona (mass per 1m² of solar surface). The corona mass is close to the mass of corona protons, their number in the same column is equal to

$$n = \frac{\mu}{m_p} = \frac{pR^2}{GMm_p}.$$

Here m_p is the proton mass. The number of electrons will be the same, and each of them can be considered as the metal ball with radius r , absorbing the radiation and emitting it in the random direction. The part of radiation, scattered by all electrons in the column is the ratio of total square of all electron disks and square of the column (which is equal to unity):

$$\tau = n \cdot \pi r^2 = \frac{\pi p R^2 r^2}{GMm_p} = 4 \cdot 10^{-7}.$$

Such part of solar emission is observed as corona emission. The magnitude of the corona is equal to:

$$m = m_0 - 2.5 \lg \tau \approx -11.$$

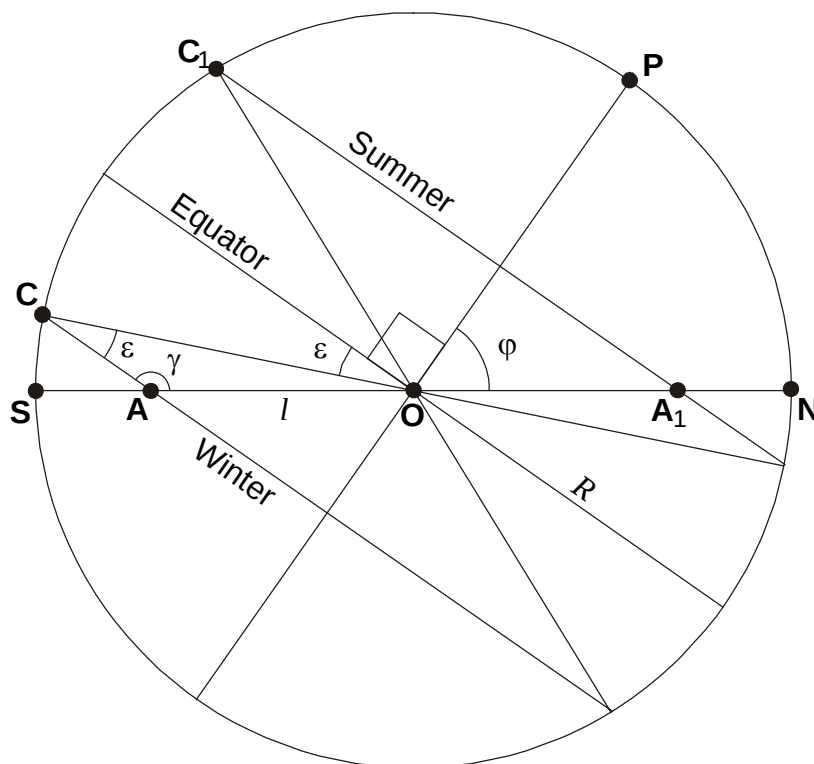
Here m_0 is the Sun magnitude. It is quite good estimation, being close to real value. Actually the corona brightness decreases due to partial occultation by the Moon, but this effect is not sufficient, since the basic contribution to the brightness is made by tangential corona regions. From the other side, the corona brightness increases owing to other emission mechanisms (forbidden lines of heavy ions) and owing to decrease of g and size increase of the outer part of the corona.

RUSSIAN OPEN SCHOOL ASTRONOMICAL OLYMPIAD BY CORRESPONDENCE – 2007

PROBLEMS WITH SOLUTIONS

1. Problem. Having observed the sunrise every day in the same location, the astronomer noticed that the azimuth of the sunrise point changes in the range of 90° during the year. Please find the latitude of the observation place. The refraction and solar disk size can be neglected. (*E.N. Fadeev*)

1. Solution. Let's consider the northern hemisphere of the Earth and draw the celestial sphere projected on the plane of celestial meridian:



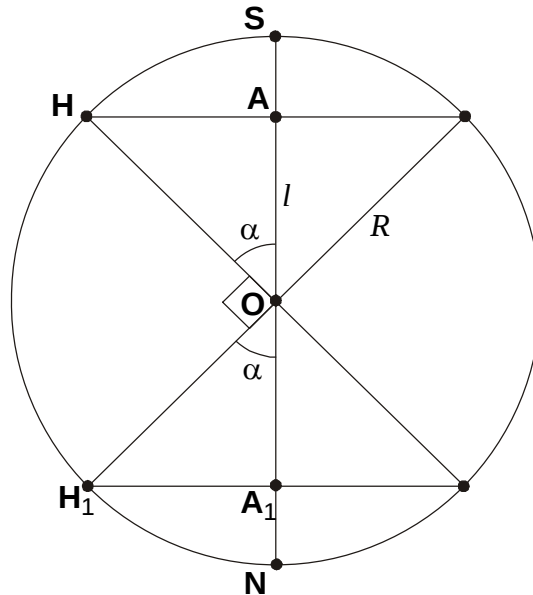
Here **P** is the North Pole of the sky, **S** is the south horizon point, **AC** and **A₁C₁** are the projections of the daily path of the Sun above the horizon at the winter and summer solstice days. Since **AC** is parallel to the equator projection, the angle **OAC** is equal to

$$\gamma = 90^\circ + \varphi,$$

where φ is the latitude of the observation place. The angle **ACO** is equal to the angle between equator and ecliptic, ε , which value is 23.4° . Using the sine theorem, we write the relation of the length **OA** and the celestial sphere radius:

$$\frac{l}{\sin \varepsilon} = \frac{R}{\sin \gamma} = \frac{R}{\cos \varphi}.$$

From the symmetry of the picture, the length **OA₁** for the summer solstice is also equal to l . Let us look to the celestial sphere from the zenith. The points **A** and **A₁** are the projections of the sunrise points **H** and **H₁** at the winter and summer solstice on the noon line.



Since the angle $\mathbf{HOH}_1$ is equal to 90° , the angle α , which is equal to the module of the azimuth of winter solstice sunrise point is equal to 45° . From the previous equation, we express the latitude value:

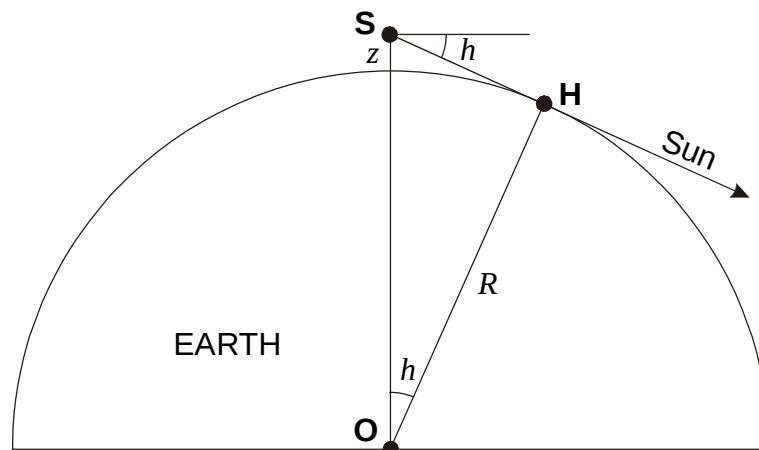
$$\varphi = \arccos\left(\sin \varepsilon \frac{R}{l}\right) = \arccos\left(\frac{\sin \varepsilon}{\cos \alpha}\right) = \arccos(\sqrt{2} \sin \varepsilon) = \pm 55.8^\circ.$$

Second value corresponds to the southern hemisphere, this case is analogous to the first one.

2. Problem. The traveller came to the equator on the vernal equinox day. At the sunset moment he starts to climb on the northern slope of the mountain inclined to the horizon by the angle 10° . He does it to see the centre of the solar disk on the horizon exactly and continuously. During what time will he succeed in doing this if he can move with the velocity up to 5 m/s? The relief surrounding the mountain and the refraction can be neglected. (*O.S. Ugolnikov*)

2. Solution. The azimuth of the Sun is not changing near the sunset at the equator, and the horizontal component of the traveller's velocity directed southwards does not change the conditions of Sun observations. The vertical shift will cause the horizon depression effect, and the centre of the solar disk will be seen for a little time after the sunset near the mountain.

Let's find the relation between the Sun depression below the mathematical horizon h and the altitude z where the centre of the disk will be seen at the horizon (see the figure). The observer is situated in the point **S**. From the triangle **SHO** we see



$$R = (R + z) \cos h.$$

Here R is the radius of Earth. Taking into account that the angle h is small and its cosine is close to unity, we obtain the expression for the altitude z :

$$z = R \cdot \left(\frac{1}{\cos h} - 1 \right) \approx R \cdot (1 - \cos h) \approx \frac{R h^2}{2},$$

where h is expressed in radians. The accuracy of this formula is enough for the problem considered. To reach this altitude, the observer must cover the following distance on the slope of the mountain:

$$l = \frac{z}{\sin \alpha} = \frac{R h^2}{2 \sin \alpha},$$

where α is the angle of the mountain slope. The observer rotates with the Earth in the plane perpendicular to the figure plane and sees the perpendicular depression of Sun under the horizon. This depression depends on time as

$$h = \omega \cdot (t - t_0),$$

where t_0 is the sunset and traveller's motion start moment, ω is the synodic angular velocity of the Earth:

$$\omega = \frac{2\pi}{T} = 7.272 \cdot 10^{-5} \text{ s}^{-1}.$$

Here T is the solar day duration. From last expressions we obtain the dependency of distance to go on the time after the sunset:

$$l = \frac{R \omega^2 (t - t_0)^2}{2 \sin \alpha}.$$

To see the centre of the solar disk on the horizon, the observer must move with constant acceleration

$$a = \frac{R \omega^2}{\sin \alpha}.$$

The value of this acceleration is 0.194 m/s^2 , that is possible for the human climbing the mountain for the definite time. The value of this time is

$$t = \frac{v}{a} = \frac{v \sin \alpha}{R \omega^2}$$

or 25.7 seconds. Note that in mid-latitudes, where Sun depresses under horizon more slowly, it is possible to hold it on the horizon climbing the mountain for the several minutes.

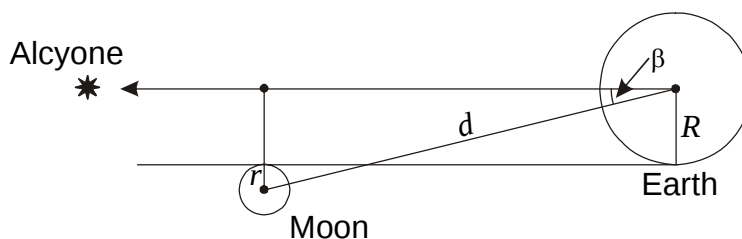
3. Problem. Astronomers know the Metonic cycle of eclipses alongside with Saros for a long time. The Metonic cycle contains 254 sidereal months being very close to 19 tropical years. Owing to this, the cycle can be used to predict not only the eclipses, but also the lunar occultations of stars. Every Metonic cycle the consequence of occultations is almost the same. In 19 years after the occultation of the star Alcyone (η Tauri, the brightest star of Pleiades cluster) the similar one can occur. How many Metonic cycles in a consequence can contain the similar Alcyone occultation? The durations of sidereal and draconic months are equal to 27.321662 and 27.212221 days, respectively. The ecliptic latitude of Alcyone is equal to $+4^\circ 03'$. (O.S. Ugolnikov)

3. Solution. Having compared Saros and Metonic cycles for the eclipses, it must be mentioned that Metonic cycle is less exact and, as 19 years have gone, solar or lunar eclipse sufficiently changes its characteristics. The duration of Metonic sequence for the eclipses is less than the one for Saros cycle. Metonic cycle was mentioned owing to its unique property – the affinity of the duration to the integer number of tropical years (the duration is 19.000275 tropical years or 18.999538 sidereal years). Owing to this, Metonic cycle can be used not only for the eclipse predictions, but also for ones for lunar occultations of stars. The accuracy of such predictions depends on the fact, how close the number of draconic months in the cycle to the integer value. Having denoted the sidereal and draconic months as T_S and T_D , we calculate the number of draconic months in Metonic cycle:

$$N_D = \frac{254 \cdot T_S}{T_D} = 255.0215.$$

It means that during one Metonic cycle the Moon completes 255 revolutions respectively the node line (the crossing of lunar orbit and ecliptic planes) and also makes 0.0215 revolutions or moves by 7.75° . It is the change value of the distance of the Moon and the star being occulted from the lunar orbit node after 19 years. We denote this angle as γ .

The ecliptic latitude of Alcyone ($+4^\circ 03'$) is close to the value of lunar orbit inclination, i ($5^\circ 09'$). Differing from the eclipses and star occultation near the ecliptic, the occultations of Alcyone can take place far from lunar orbit nodes, when the Moon moves northwards from ecliptic. This situation takes place in 2007, for example. Let us find the range of values of ecliptic latitude of the Moon, when it can occur Alcyone.



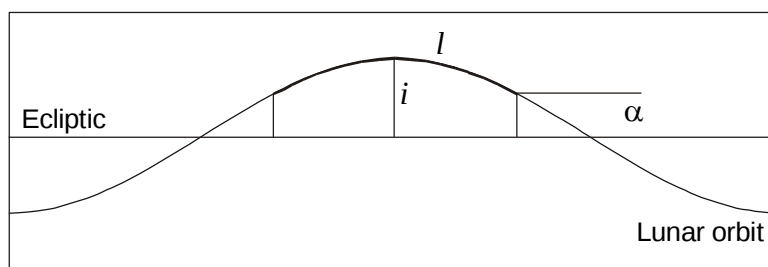
We see that maximal geocentric angular distance between the Moon and Alcyone for the occultation to be observed is equal to

$$\beta = \arcsin \frac{R+r}{d} = 1.209^\circ$$

or $1^\circ 13'$. Taking into account that the Moon moves by the little angle to the ecliptic, we obtain the values of minimal and maximal ecliptic latitudes of the Moon:

$$+4^\circ 03' - 1^\circ 13' = +2^\circ 50'; \quad +4^\circ 03' + 1^\circ 13' = +5^\circ 16'.$$

The maximal value is higher than inclination of lunar orbit and can't be reached. Let's find the angular length of the part of lunar orbit, where the ecliptic latitude is higher than $+2^\circ 50'$ (we denote this angle as α).





Since the angle i is small, we can assume that the ecliptic latitude changes along the orbit by sinusoidal law. The length of arc l , where the latitude is higher than α , is equal to

$$l = 2 \arccos \frac{\alpha}{i} = 113.2^\circ.$$

Each Metonic cycle the Moon and the star will shift to the left by this arc. The total number of occultations in one sequence will be

$$N = \frac{l}{\gamma} = 14.6.$$

Real number of occultations can be equal to 14 or 15, and the total duration of one sequence is 247 or 266 years. It is much higher than the one for eclipses or near-ecliptic occultations. Since the Metonic cycle contains integer number of years, the occultations will be observed at the same phase of the Moon and in the same or nearby dates. “New Year” occultation of Alcyone at December, 31st, 2006 is the one in the sequence of 14 occultations from January, 1st, 1931 to January, 1st, 2178, each one occurs near the New Year.

4. Problem. Large telescope of future generation is used for the visual observations of artificial minor planet – ideal mirror metal ball with diameter equal to the telescope lens diameter. The ball moves around the Sun by the circular orbit with radius equal to 3 a.u. Please find the minimal value of lens diameter. The sky background and influence of atmosphere can be neglected. (*O.S. Ugolnikov*)

4. Solution. Let us denote the solar luminosity as L , and minimal light flux seen by the naked eye as j (it corresponds to the 6^m star). The solar energy flux near the metal ball is equal to

$$J_1 = \frac{L}{4\pi d^2},$$

where d is the distance between the Sun and asteroid. All energy falling to the ball is reflecting isotropically. The ball is emitting with the following luminosity:

$$l = J_1 \cdot \pi R^2 = \frac{L R^2}{4d^2}.$$

The ball brightness on the Earth is maximal during the opposition. The energy flux from the ball will reach the value:

$$j_1 = \frac{l}{4\pi (d - d_0)^2} = \frac{L R^2}{16\pi d^2 (d - d_0)^2}.$$

Here d_0 is the radius of Earth’s orbit (the astronomical unit). If the eyepiece magnification is chosen correctly and there is no handicaps than the eye with radius r will catch the whole energy collected by the objective with radius R (the same as the metal ball), and the flux incoming to an eye, will be

$$j = j_1 \cdot \frac{R^2}{r^2} = \frac{L R^4}{16\pi r^2 d^2 (d - d_0)^2}.$$

This flux must be not less than the flux from the 6^m star. The solar flux on the Earth is equal to

$$J = \frac{L}{4\pi d_0^2} = C \cdot j,$$

where the constant C can be calculated from the known value of Sun magnitude m_0 (-26.8):

$$C = 10^{0.4(m-m_0)} = 1.32 \cdot 10^{13}.$$

Using last three formulae, we express the minimal radii of the ball and telescope lens:

$$R = \sqrt{\frac{2rd(d-d_0)}{d_0\sqrt{C}}}.$$

The numerical value is 40 meters, the diameter is equal to 80 meters. There is no such telescope in the present time, but it is possible to appear in the nearest future.

5. Problem. Two stars have the same physical parameters. They are observed close to each other in the sky, but their distances are different. Both stars and the observer are situated inside the uniform cloud of interstellar dust. The photometric measurements of these stars in B band gave the results 11^m and 17^m , in V band the results were 10^m and 15^m . What is the ratio of distances to these stars? Assume that the extinction property of interstellar dust is proportional to the wavelength in the degree of (-1.3) . (*E.N. Fadeev, O.S. Ugolnikov*)

5. Solution. Dust absorption along the emission path from the source to the observer decreases the brightness of the source. The energy flux registered by the observer is equal to J instead of J_0 :

$$J = J_0 \cdot e^{-k \cdot r},$$

where r is the length of the emission path inside the dust cloud, k is the absorption coefficient, proportional to the dust density and defined by the dust properties. If the source and observer are situated inside the homogenous dust cloud, then r will be the distance between them. If r is expressed in parsecs and k is expressed in parsecs^{-1} , we can write the relations between visible (m) and absolute (M) magnitudes of the star with account of dust absorption:

$$m = M - 5 + 5 \lg r + E \cdot r,$$

where $E = 1.086 \cdot k$.

The coefficients k and E depend on the wavelength. The relation of coefficients E for the photometric bands B (effective wavelength λ_B is equal to 4400 Å) and V (effective wavelength λ_V is equal to 5500 Å) is following:

$$\frac{E_B}{E_V} = \left(\frac{\lambda_B}{\lambda_V} \right)^{-1.3} = 1.3365.$$

Wavelength dependency of the absorption changes the color of the star, making it redder. The change of color index B–V (difference of magnitudes in two bands) per the distance unit is proportional to the absorption coefficient in V band:

$$\frac{E_{B-V}}{E_V} = \frac{E_B - E_V}{E_V} = 0.3365 = \frac{1}{2.97}.$$

In the absence of absorption the color index would not depend on the distance. Taking the expressions of magnitudes in B and V bands and subtracting the second relation from the first, we obtain

$$m_B - m_V = B - V = (M_B - M_V) + E_{B-V} \cdot r,$$

Let us denote the distances to the near and far stars as r_1 and r_2 . Since these stars are the same, their absolute magnitudes are equal. The difference of color indexes is following:

$$(B - V)_2 - (B - V)_1 = E_{B-V} \cdot (r_2 - r_1) = 1^m,$$

thus,

$$E_V \cdot (r_2 - r_1) = 2.97^m.$$

The difference of stellar magnitudes of the stars in V band is equal to

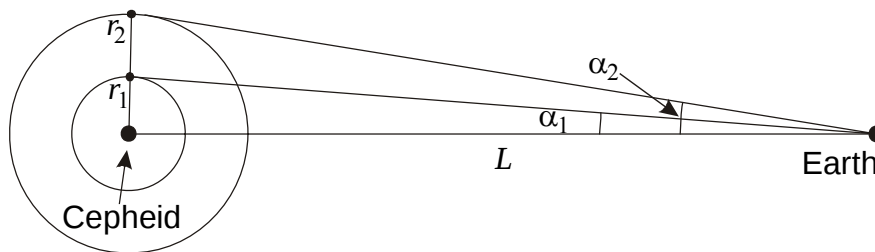
$$V_2 - V_1 = 5 (\lg r_2 - \lg r_1) + E_V \cdot (r_2 - r_1) = 5^m.$$

From last two equations we obtain

$$5 \lg \frac{r_2}{r_1} = 2.03; \quad \frac{r_2}{r_1} = 2.55.$$

6. Problem. The Cepheid variable star with period equal to 50 days is visible by the naked eye. Having observed it with the telescope, astronomers detected two-layers reflecting nebula around this star. The nebula scatters the Cepheid emission. Layers' angular radii are equal to $10''$ and $21''$. The brightness of both layers changes with the same period equal to 50 days, reaching the maximum in 30 and 18 days after the Cepheid maximum for inner and outer layer, respectively. Please find the distance to the Cepheid. (*E.N. Fadeev, O.S. Ugolnikov*)

6. Solution. Spherical nebula surrounding the Cepheid is reflecting the light from the stars into the space. As for the spherical planetary nebulae, the brightness of the reflecting nebula reaches the maximum at the edge, where the line of sight passes by the long tangent path through the nebula. The emission maximum of the nebula occurs when the Cepheid maximum is observed from the edge of this nebula.



Since the amounts of time of light propagation from the Cepheid and the edge of both layers are the same, the time difference between maxima of Cepheid and layer with number i is following:

$$\Delta T_i = \frac{r_i}{c}.$$

But this value can exceed the Cepheid period. In general case, it will be related with measured values Δt_i by the next formulae:

$$\Delta T_i = n_i \cdot T + \Delta t_i = T \cdot (n_i + \Delta \phi_i), \quad \Delta \phi_i = \Delta t_i / T.$$

The values Δt_i are not less than zero and less than Cepheid period T (the delay phases $\Delta \phi_i$ are not less than zero and less than unity). The integer non-negative numbers n_i are unknown. It is the basic problem of the method of measurement of the distances by the observations of reflecting nebulae.

Expressing the layer radius r_i from its angular size and distance, we obtain

$$T \cdot (n_i + \Delta\varphi_i) = \frac{L \alpha_i}{c}.$$

Converting the period value into days ($8.64 \cdot 10^4$ seconds), the distance – into kiloparsecs ($3.086 \cdot 10^{19}$ meters) and angular radius of the layers – into angular seconds ($4.848 \cdot 10^{-6}$ radians), we rewrite this formula as

$$T_d \cdot (n_i + \Delta\varphi_i) = 5.77 \cdot L_{Kpc} \alpha_i''.$$

To find the distance L , we have to know the numbers n_i . We will be helped by the fact that we see two layers. Having written last formula for both layers and dividing the first one on the second, we see

$$\frac{n_2 + \Delta\varphi_2}{n_1 + \Delta\varphi_1} = \frac{\alpha_2''}{\alpha_1''} = 2.1.$$

The delay values of 30 and 18 days correspond to the phases 0.60 and 0.36. Having taken the value $\Delta\varphi_1$ (0.60) and different n_1 , we define the values of n_2 и $\Delta\varphi_2$ and choose the ones when $\Delta\varphi_2$ is equal to observed value (0.36):

n_1	$(n_1 + \Delta\varphi_1)$	$2.1 \cdot (n_1 + \Delta\varphi_1)$	n_2	$\Delta\varphi_2$	L , kps
0	0.60	1.26	1	0.26	1.39
1	1.60	3.36	3	0.36	
2	2.60	5.46	5	0.46	
...					
11	11.60	24.36	24	0.36	10.05
...					
21	21.60	45.36	45	0.36	18.72
...					

As we see in the table, there are many solutions meeting the observed data. Last column contains the corresponding values of distance to the star. To choose the correct one, we remember that the Cepheid is seen by the naked eye. Cepheids are the bright supergiants, their absolute magnitudes are related with the period, the value for our star (M) is close to -6 . If there is no absorption, than the visible magnitude is equal to

$$m = M - 5 + 5 \lg L_{Kpc}.$$

For the minimal possible distance to the star (1.39 kps) the magnitude will be close to 5^m , that is enough for the naked eye observations. Second distance value (about 10 kps) leads to the magnitude 11^m . In fact, the star will be even fainter due to the absorption, which becomes strong at such distances. The same can be said about all other possible distance values. Finally, taking into account all available data, we come to the only solution: the distance to the Cepheid is equal to 1.39 kps.

7. Problem. The Planck unit system is often used in astrophysics and cosmology. In this system the gravitation constant G , light velocity c and Planck constant h are equal to unity and have no dimension. Using this system, we can express any physical value in units of any other physical value. Please, use the Planck unit system to express the astronomical unit (the distance between Earth and Sun) in seconds, kilograms and Joules. Do these numbers have the physical sense? (*N.I. Perov*)

7. Solution. Let us start from the calculations of Planck units of length, mass and time in traditional unit system (SI). These values are the combinations of three physical constants – G , c , and h with dimension of length, mass and time (for the thermodynamic units the Boltzman constant, k , is added). It can be done using the dimensions method. The dimensions of physical constants are following:

$$\begin{aligned}\langle G \rangle &= L^3 M^{-1} T^{-2}; \\ \langle c \rangle &= L^1 T^{-1}; \\ \langle h \rangle &= L^2 M^1 T^{-1}.\end{aligned}$$

Here L , M и T are the dimensions of length, mass and time, respectively. The Planck length is expressed as

$$l_P = G^\alpha c^\beta h^\gamma,$$

where the power degrees meet the following equations, respecting to the length, mass and time:

$$\begin{aligned}3\alpha + \beta + 2\gamma &= 1, \\ -\alpha + \gamma &= 0, \\ -2\alpha - \beta - \gamma &= 0.\end{aligned}$$

Having solved these equations, we obtain $\alpha = \gamma = 1/2$, $\beta = -3/2$. Finally,

$$l_P = \sqrt{\frac{Gh}{c^3}}.$$

The Planck unit of length is turned out to be equal to $4 \cdot 10^{-35}$ meters. Analogously, we express the Planck time and Planck mass:

$$t_P = \sqrt{\frac{Gh}{c^5}}; \quad m_P = \sqrt{\frac{hc}{G}}.$$

The numerical values are 10^{-43} seconds and $5 \cdot 10^{-8}$ kilograms, respectively. These values have the physical sense. Planck length and Planck time are characterizing the earliest stage of Universe, where existing physical theories can not be applied. The Planck mass (it is also called “maximon mass”) is the upper limit of the mass of elementary particle.

In the Planck unit system all these values have no dimension, being equal to unity. Usual meter, kilogram and second are also dimensionless, their numerical values are reverse to the values of Planck length, mass and time in SI system. For example, one meter is corresponding to the number $2.5 \cdot 10^{34}$. Astronomical unit, expressed in Planck units, will have the value

$$\langle D \rangle = 1.5 \cdot 10^{11} \cdot 2.5 \cdot 10^{34} = \frac{D}{l_P} = Dc \sqrt{\frac{c}{Gh}} = 3.75 \cdot 10^{45}.$$

To express the unit **A** in terms of unit **B** in Planck system, we have to calculate the ratio of units **A** and **B** in this unit system. To express the astronomical unit in seconds, we divide it on the Planck expression of second or multiply it on the Planck time:

$$\langle D \rangle_T = \langle D \rangle \sqrt{\frac{Gh}{c^5}} = Dc \sqrt{\frac{c}{Gh}} \sqrt{\frac{Gh}{c^5}} = \frac{D}{c} = 500 \text{ s}.$$

The physical sense of this value is seen in the formula: it is the time the light need to cover the distance equal to one astronomical unit. Than we express the astronomical unit in kilograms:

$$\langle D \rangle_M = \langle D \rangle \sqrt{\frac{hc}{G}} = Dc \sqrt{\frac{c}{Gh}} \sqrt{\frac{hc}{G}} = \frac{Dc^2}{G} = 2 \cdot 10^{38} \text{ kg}.$$

With the accuracy of factor 2 it is the mass of black hole with the radius equal to astronomical unit. To express the astronomical unit in Joules, we start from calculation of Planck energy:

$$E_P = l_P^2 m_P t_P^{-2} = \sqrt{\frac{hc^5}{G}},$$

that is equal to $5 \cdot 10^9$ Joules. The expression of astronomical unit in Joules is following:

$$\langle D \rangle_E = \langle D \rangle \sqrt{\frac{hc^5}{G}} = D c \sqrt{\frac{c}{Gh}} \sqrt{\frac{hc^5}{G}} = \frac{D c^4}{G} = 2 \cdot 10^{55} \text{ Joules}.$$

By the order of value it is the total mass energy of the black hole with radius equal to one astronomical unit.

The problems about comets evolution in the Solar System

8. Problem. Approximately once in 5 years people on the Earth can observe bright comets, whose nuclei have the radius about one kilometer. The orbits of such comets are close to parabolic. Assuming that these nuclei are uniformly filling the volume of spherical Oort cloud with radius 10000 a.u., estimate the number of large comet nuclei and mass of Oort cloud. (O.S. Ugolnikov)

8. Solution. This problem requires only the estimation with the accuracy of value order. It can be solved using the assumptions simplifying the model and calculations. Depending on plausibility of the model, we will obtain the answer, more or less approached to reality. Some of these assumptions (the same nuclei sizes, their uniform distribution in the cloud) are already made in the problem text, that must be taken into account while solving it.

The simplest model of solution is built on the assumption that all comets are passing by the Sun with some velocity by the straight lines. The comets those come to the inner part of Solar System (closer than the giant planet Jupiter with orbit radius L equal to $8 \cdot 10^{11}$ meters) strongly deviate from their path, come to the region of Earth-type planets, increase their brightness and can be observed from Earth as bright comets.

The frequency of such events depends on the nuclei concentration in the Oort cloud and their typical velocity relatively the Sun. Of course, the comet velocity rapidly increases during the flyby near the Sun, but here we mean the comets velocity far away from the Sun, defining the frequency of bright comet occurrence. Since the Oort cloud is stable, we can assume that order of this velocity is the same with circular velocity at the distance R equal to the Oort cloud radius:

$$v = \sqrt{\frac{GM}{R}}.$$

Here M is the solar mass. This velocity is equal to 300 m/s. Since the nuclei distribution is uniform, the probability of comet flyby does not depend on the direction and we can assume that all bodies fly with the velocity v in the same direction or the Sun moves through motionless Oort cloud with the same velocity. During the time t equal to 5 years or $1.5 \cdot 10^8$ seconds the Sun will cover the path $v \cdot t$ and the inner parts of Solar system will draw the cylinder with the volume

$$V = \pi L^2 v t.$$

According to the problem, one comet will fly near the Sun during this period. Thus, the nuclei concentration in the Oort cloud is the unity per this volume:

$$n = \frac{1}{V} = \frac{\sqrt{R}}{\pi L^2 t \sqrt{GM}}.$$

Multiplying this concentration on the total volume of Oort cloud, we estimate the number of large nuclei in the cloud:

$$N = \frac{4}{3}\pi R^3 n = \frac{4R^3}{3L^2 vt} = \frac{4R^{7/2}}{3L^2 t \sqrt{GM}} \approx 10^{11}.$$

To estimate the mass of one nucleus, we assume that its density is close to the water density (which is close to reality):

$$m = \frac{4}{3}\pi r^3 \rho.$$

Here r is the nucleus radius. The nucleus mass is equal to $4 \cdot 10^{12}$ kg, the total mass of all kilometer-sized nuclei in the Oort cloud is turned out to be equal to $10^{23} - 10^{24}$ kg, or slightly less than the mass of the Earth. In fact, we had not taken into account the small bodies, which number can be very large. By recent estimations, the mass of Oort cloud is comparable with mass of Jupiter (10^{27} kg).

There are some other methods to estimate the number of comet bodies in Oort cloud. Let's assume that all bodies are rotating around the Sun by the orbits close to circular. The velocity of such revolution, v , was calculated above (300 m/s). To change the orbit and go to the inner part of Solar System, the nucleus must interact with the other one. The gravitational field of comet body is weak, the escape velocity is less than the relative velocities of nuclei. Thus, the orbit can be changed only during the collisions of the nuclei. Analogous to the calculations above, we express the probability of collision of one body with any other body during the period t :

$$p = n \cdot \pi r^2 \cdot v t.$$

However, such event can occur with any nucleus. To determine the total number of collisions during this period, we multiply this probability on the number of nuclei:

$$N_C = p \cdot n \cdot \frac{4}{3}\pi R^3 = \frac{4}{3}\pi^2 n^2 r^2 R^3 vt.$$

If one collision occurs during the time t equal to 5 years or $1.5 \cdot 10^8$ seconds, than the concentrations of comet bodies in the Oort cloud is equal to

$$n = \frac{1}{2\pi r R} \sqrt{\frac{3}{Rvt}},$$

and the total number of nuclei is following:

$$N = \frac{4}{3}\pi R^3 n = \frac{2R}{r} \sqrt{\frac{R}{3vt}} = 5 \cdot 10^{14},$$

It is leading us to the total mass of Oort cloud close to the mass of Jupiter – 10^{27} kg.

Such strong difference of results obtained by two methods is not surprising, since both methods are simplified models. But these estimations give the evident performance of Oort cloud. Which model is more exact – it depends on real sizes and velocity distribution of the comet bodies in the Oort cloud.

9. Problem. The comet with parabolic orbit comes to perihelion, approaching close to Jupiter. After the gravitational interaction with giant planet the comet comes to the new heliocentric orbit with the period equal to the half of Jupiter's orbital period. Please find the angle of comet's turn in the gravitational field of Jupiter. Consider the orbit of Jupiter as circular, the orbit planes of comet and Jupiter are the same. (*N.I. Perov*)

9. Solution. Evolution of the comet orbit can be considered as follows: during the period of approaching to Jupiter (that is many times shorter than the orbital period of the planet) the comet is the temporal satellite of Jupiter, moving by the hyperbolic orbit. Before and after that the comet is the satellite of the Sun. According to the energy conservation law, the planetocentric velocity before and after approach is the same, only its direction is changing. But it changes the heliocentric velocity of the comet and its orbit. It is the passive gravitational maneuver, during which the comet transits from the parabolic orbit to elliptical one. The reverse maneuver is also used by spacecrafts, going to the outer parts of Solar System or even escaping from there. It meets the energy conservation law, since the additional energy is been taking (or giving) by the planet, also slightly changing its orbit.

The orbit of Jupiter is circular, the comet orbit is parabolic. Their heliocentric velocities are equal to

$$v_P = \sqrt{\frac{GM}{R}}, \quad v_C = \sqrt{\frac{2GM}{R}}.$$

The comet reaches perihelion of its old orbit, and its velocity (as the one of Jupiter) is directed perpendicular to the radius-vector. Since the orbital planes coincide, the velocities are parallel. They can be directed to the same side or to opposite sides. Let's consider the second case. The planetocentric velocity of comet is following:

$$u_* = v_C + v_P = \sqrt{\frac{GM}{R}}(\sqrt{2} + 1).$$

After the approach it will have the same module. New heliocentric velocity v_* is the vector sum of velocities u_* and v_P . Its module cannot be less than the difference of these velocities:

$$\mathbf{v}_* = \mathbf{u}_* + \mathbf{v}_P, \quad v_* \geq |u_* - v_P| = \sqrt{\frac{2GM}{R}}.$$

Independently on the angle of comet's turn it will be on the opened orbit and escape the Solar System, which contradicts with the problem conditions. Thus, comet moves in the same direction with Jupiter, and its planetocentric velocity is equal to

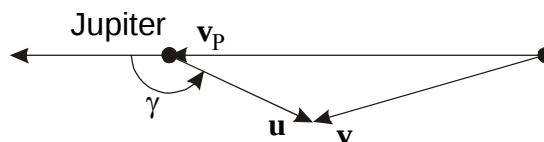
$$u = v_C - v_P = \sqrt{\frac{GM}{R}}(\sqrt{2} - 1)$$

and remains the same after the approach. To find the new heliocentric velocity, we calculate the major semi-axis of new comet orbit a , comparing it with the orbit of Jupiter:

$$\frac{a^3}{R^3} = \frac{T_C^2}{T^2} = \frac{1}{4}, \quad a = \frac{R}{\sqrt[3]{4}}.$$

Heliocentric comet velocity after the approach to Jupiter is equal to

$$v = \sqrt{GM \cdot \left(\frac{2}{R} - \frac{1}{a} \right)} = v_P \sqrt{(2 - 2^{2/3})}.$$



Let us denote the angle of comet's turn in the gravitational field of Jupiter (the turn of vector of comet velocity $\mathbf{u}$ relatively Jupiter) as γ . This angle is adjacent to the one in the triangle, all its sides are known. The angle required is calculating using the cosine theorem:

$$\gamma = \arccos \frac{v^2 - u^2 - v_P^2}{2uv_P} = \arccos \left(1 - \frac{\sqrt{2} + 1}{\sqrt[3]{2}} \right) = 156.4^\circ.$$

10. Problem. At May, 16th, 2006, Earth passed by the fragments of comet Schwassmann-Wachmann 3. Being observed from the Earth, the cluster of fragments had the string-type form with angular length equal to 40° , the distances to the edges of this string were equal to 0.055 and 0.105 a.u. Assuming that the comet Schwassmann-Wachmann 3 had broken up by the momentary isotropic explosion near the perihelion in October, 1995, estimate the fragments scattering velocity during the explosion. In what time the meteor shower created by this comet will be annually observed from the Earth? The comet perihelion distance is equal to 0.939 a.u., the orbit eccentricity is equal to 0.693. (O.S. Ugolnikov)

10. Solution. It can seem strange, that after isotropic explosion the fragment situated in the sky and in the space as string-type cluster. But it can be easily explained. Such explosions happen to comets near their perihelion, and the comet Schwassmann-Wachmann 3 is not the exclusion. Little additional velocity directed perpendicular to the orbital one will not cause the change the orbital period, and all the fragments pushed in the side directions will be in the same point after the orbital revolution. Their distance from the center of the cluster will remain quite little for a long time.

But if the additional velocity is directed along (or towards) the orbital motion, it will change the orbital period. Each orbital revolution this fragments will be situated farther from the cluster center. Note that the fragments pushed straight during the explosion, will appear behind the cluster after the revolution. Finally, the cluster will be stretched along the orbit, forming the meteor stream that will be observed from the Earth if it crosses the orbit of former comet. The stream length will increase in the perihelion and decrease in aphelion. Large angular size of the cluster of comet Schwassmann-Wachmann 3 fragments after only 11 years from the explosion is caused by the close approach to the Earth.

The spatial length of the cluster can be calculated using the given data:

$$l = \sqrt{d_1^2 + d_2^2 - 2d_1d_2 \cos\theta} = 0.072 \text{ a.u.}$$

or 10.8 mln km. Here d_1 and d_2 are the distances to the edges of the string, θ is the angular length of the string. The major semi-axis of the orbit is equal to

$$a = \frac{p}{1-e} = 3.059 \text{ a.u.}$$

Here p is the perihelion distance, e is the orbit eccentricity. According to the third Kepler law the orbital period of the comet is equal to 5.35 years, and comet completed two orbital revolutions after the explosion.

The heliocentric velocity of the comet near the Earth (at the distance r from the Sun) can be calculated by the formula

$$v = \sqrt{GM \cdot \left(\frac{2}{r} - \frac{1}{a} \right)} = v_0 \sqrt{2 - \frac{r}{a}}.$$

Here M is the solar mass, v_0 is the orbital velocity of the Earth. The numerical value of comet velocity is 38.5 km/s. Forward part of the cluster is moving slightly slower than the back one, but outruns it by the time

$$\tau = \frac{l}{v},$$

that is equal to $2.8 \cdot 10^5$ seconds or 3.25 days. This value is $1.66 \cdot 10^{-3}$ of orbital period of comet Schwassmann-Wachmann 3. This difference appeared after 2 orbital revolutions. Thus, the orbital period of the forward fragments of the cluster is less by $8.3 \cdot 10^{-4}$ part than the one from back fragments. After $(1/8.3 \cdot 10^{-4})$ or 1200 orbital revolutions of the cluster the forward part will made the one total revolution more than the back part and the cluster will fill the whole orbit. The Earth will be meeting this meteor stream each year, crossing its orbit. Multiplying 1200 on the orbital period, we find the required value of time, about 6400 years. It is the answer to the second question of a problem.

To find the scattering velocity, we note that the orbital period of the cluster edge differs from the one in the center by the value

$$\Delta T = \frac{8.3 \cdot 10^{-4}}{2} \cdot T = \frac{\tau}{4},$$

that is equal to $7 \cdot 10^4$ seconds. Let's consider the fragments moving in the back part of the cluster. Their perihelion velocity, orbit eccentricity and period are equal to $v_P + \Delta v_P$, $e + \Delta e$ and $T + \Delta T$, where v_P , e and T are the same values for the cluster centre. The perihelion velocity of the cluster centre is equal to

$$v_P = \sqrt{\frac{GM}{p}(1+e)} = v_0 \sqrt{\frac{r}{p}(1+e)}$$

or 40.0 km/s. Taking into account that all additions to the parameters at the cluster edge are small, the perihelion velocity of the back part can be written as

$$v_P + \Delta v_P = \sqrt{\frac{GM}{p}(1+e+\Delta e)} = v_P \sqrt{\frac{1+e+\Delta e}{1+e}} \approx v_P \cdot \left(1 + \frac{\Delta e}{2(1+e)}\right).$$

Thus,

$$\Delta e = \frac{2\Delta v_P(1+e)}{v_P}.$$

The value of major semi-axis of back fragments will be as follows:

$$a + \Delta a = \frac{p}{1-e-\Delta e} = \frac{p}{1-e} \cdot \frac{1-e}{1-e-\Delta e} \approx a \cdot \left(1 + \frac{\Delta e}{1-e}\right).$$

By the same way,

$$\Delta a = \frac{a \cdot \Delta e}{1-e} = 2a \frac{1+e}{1-e} \cdot \frac{\Delta v_P}{v_P}.$$

Finally, the orbital period of the back part of the cluster, according to the third Kepler law,

$$T + \Delta T = T \cdot \left(\frac{a + \Delta a}{a}\right)^{3/2} \approx T \cdot \left(1 + \frac{3\Delta a}{2a}\right).$$

This leads to relation

$$\Delta T = \frac{3T\Delta a}{2a} = 3T \cdot \frac{1+e}{1-e} \cdot \frac{\Delta v_P}{v_P}.$$

This formula gives the expression for the scattering velocity during the explosion:

$$\Delta v_P = v_P \frac{\Delta T}{3T} \cdot \frac{1-e}{1+e}.$$

Substituting the numerical data, we obtain the scattering velocity value: 1 m/s! The comet explosion is seemed to be sufficient event, but it did not have the catastrophic power. Our recognition is related with close approach to the cluster in May, 2006, and 16.5 times difference of the visual velocity of cluster expansion (the length divided by the time since the explosion) and real scattering velocity during the explosion.

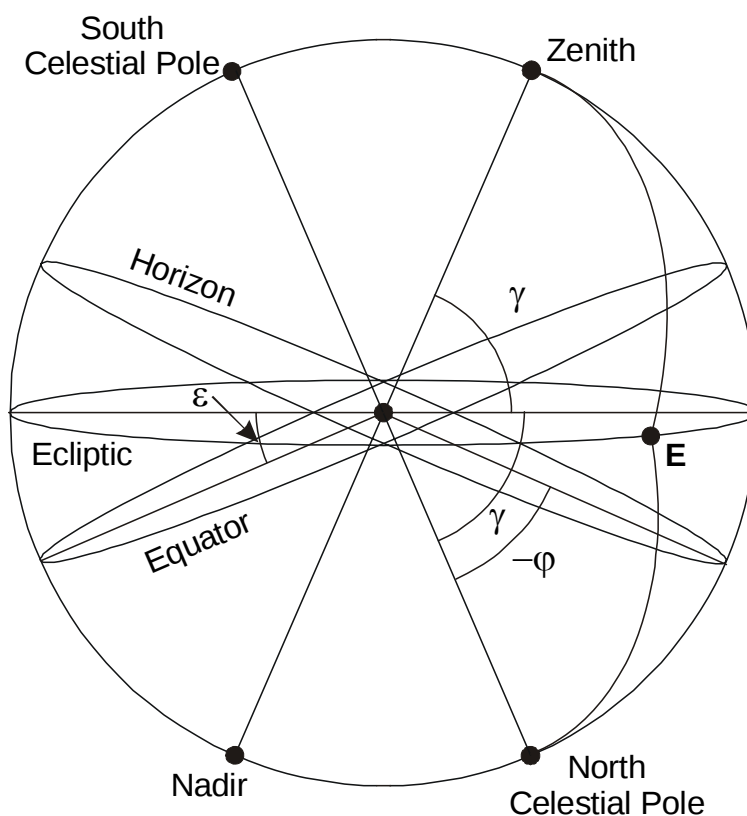
RUSSIAN OPEN SCHOOL ASTRONOMICAL OLYMPIAD BY CORRESPONDENCE – 2008

PROBLEMS WITH SOLUTIONS

1. Problem. Observer is situated in the definite point on the Earth's surface. One definite moment he noticed that each point of ecliptic had met the mysterious property: the angular distance between this point and North Celestial Pole had been equal to the zenith distance of the same ecliptic point. Disregarding the refraction, please find the latitude of the observation point. (*O.S. Ugolnikov*)

1. Solution. The conditions described above are always met on the North Pole of the Earth (latitude $+90^\circ$) where the North Celestial Pole coincides with the zenith and the angular distance from the North Celestial Pole is equal to the zenith distance not only for the points of the ecliptic, but for all points of the celestial sphere. But this is not the only solution.

If we observe somewhere far from the North Pole of the Earth, than the North Celestial Pole and the zenith are two different points of the celestial sphere. The problems conditions will be met if these two points are symmetric relatively the ecliptic line. It is seen in the figure for the example ecliptic point E.



Ecliptic is inclined to the equator by the angle ϵ equal to 23.4° . The angle between the northern polar direction (which is perpendicular to the equator) and the ecliptic plane is equal to

$$\gamma = 90^\circ - \epsilon = 66.6^\circ.$$

Symmetry of North Celestial Pole and the zenith relatively the ecliptic means that the angle between the zenith direction and the ecliptic plane is the same. The ecliptic plane must be perpendicular to the plane containing the zenith and northern polar directions. Thus, the zenith distance of North Celestial Pole is equal to

$$z_P = 2\gamma = 133.2^\circ.$$

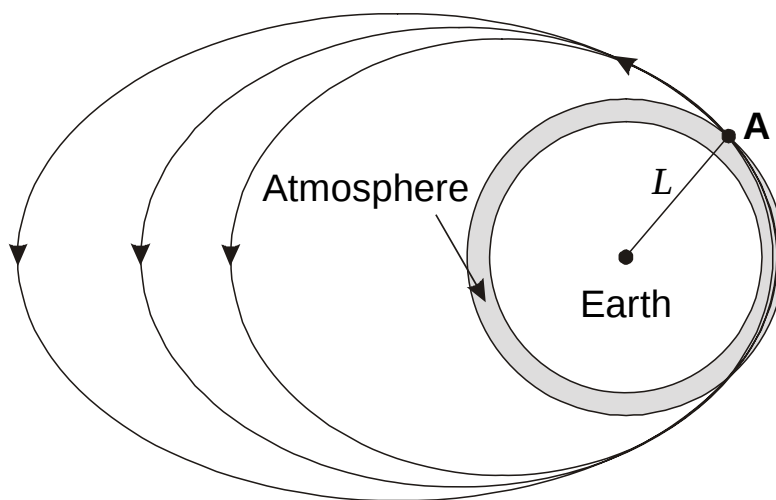
Northern Celestial Pole is below the horizon, and the observation point is in the Southern hemisphere of the Earth. The latitude is negative, its module is equal to the depth of Northern Celestial Pole:

$$\varphi = -(z_P - 90^\circ) = 90^\circ - 2\gamma = -90^\circ + 2\varepsilon = -43.2^\circ.$$

Finally, the problem condition can take place at the latitudes $+90^\circ$ (it always takes place there) and -43.2° .

2. Problem. The artificial satellite of the Earth has the mass equal to 100 kg and moves along the elliptical orbit with perigee altitude equal to 200 km and apogee altitude equal to 10000 km. Being close to perigee, the satellite is decelerated by the Earth's atmosphere. Please estimate the time, during which the satellite's orbit will become circular. The decelerating force of the atmosphere can be considered to be constant with the value 0.01 Newton, the path length of the satellite through the atmosphere each revolution is equal to the radius of the Earth. (O.S. Ugolnikov)

2. Solution. Let's explain why the orbit of the satellite will turn to circle. Being close to the apogee, the satellite is far from the Earth, it is not decelerated by the atmosphere and moves by the elliptical trajectory according to the Second Kepler law. Approaching the perigee, the velocity of the satellite increases and exceeds the circular velocity for this distance to the Earth. But here the satellite is being decelerated by the atmosphere. As we will see below, this deceleration is not too strong to lead the satellite to fall down or burn up in the atmosphere during the first revolution. But each revolution the satellite will loose the velocity and energy before the escape from the atmosphere.



Let L will be the distance from the center of the Earth to the upper border of the dense atmosphere layers, the atmosphere deceleration above this border can be neglected. The satellite velocity on this border is equal to

$$v^2 = GM \left(\frac{2}{L} - \frac{1}{a} \right),$$

where M is the Earth mass and a is orbit large semi-axis. Each revolution the values of v and a are decreasing. While the orbit is elliptical and satellite escapes from the atmosphere, its perigee altitude is changing slowly. Thus, the orbit of the satellite turns to the circle with the radius close to the perigee distance of the initial orbit. When the satellite does not escape the atmosphere, its trajectory will turn

to spiral. The satellite will go down to the Earth increasing its velocity until it burn or fall down. But this stage of orbit evolution is not considered in this problem.

The following solution is the simplified method of the calculation of the time of orbit evolution, that is quite complicated problem in general. Let's consider two consecutive moments of the satellite's escape from the dense layers of atmosphere (the point **A** in the figure). Of course, the escape points are different, but their distances from the center of the Earth, L , are the same. Let's draw the relations between the velocities of the satellite at these moments and the values of major semi-axes of the orbit at (i) and $(i+1)$ revolutions:

$$v_i^2 = GM \left(\frac{2}{L} - \frac{1}{a_i} \right),$$

$$v_{i+1}^2 = GM \left(\frac{2}{L} - \frac{1}{a_{i+1}} \right),$$

According to the constant energy law,

$$\frac{m}{2}(v_{i+1}^2 - v_i^2) = -F \cdot D,$$

Here m is the satellite mass, F is the deceleration force, D is the length of the satellite path through the atmosphere. According to the problem condition, the values of F and D are constant. Their multiplication is equal to 64 kJ, that is many times less than the kinetic energy of the satellite in perigee. Thus, the orbit will turn to circle slowly, during the large number of revolutions. The change values of major semi-axis Δa_i and orbital period ΔT_i are many times less than the values a_i and T_i themselves.

From the formulae above we obtain:

$$\frac{2FD}{GMm} = \frac{1}{a_{i+1}} - \frac{1}{a_i} \approx \frac{-\Delta a_i}{a_i^2}; \quad \Delta a_i = -\frac{2FD a_i^2}{GMm}.$$

Major semi-axis and orbital period are related with each other by the III Kepler law:

$$a_i^3 = T_i^2 \cdot \frac{GM}{4\pi^2}.$$

Mean change value of major semi-axis during (i) -revolution is the value of Δa_i divided by orbital period:

$$\psi_{i1} = \frac{\Delta a_i}{T_i} = -\frac{FD\sqrt{a_i}}{\pi m\sqrt{GM}}.$$

We can assume, that the change of major semi-axis in time will have a power law, we can find the number n , for which:

$$\psi_{in} = \frac{\Delta(a_i^n)}{T_i} = \text{const}_i.$$

Our aim is to find the number n . The parameter in the numerator of the last formula is equal to

$$\Delta(a_i^n) = a_{i+1}^n - a_i^n = (a_i + \Delta a_i)^n - a_i^n \approx n\Delta a_i a_i^{n-1}.$$

Here we had used the mathematical property of small value p :

$$(1 + \rho)^n \approx 1 + n\rho.$$

Finally, the change of value a_i^n per unit of time is equal to

$$\psi_{in} = \frac{\Delta(a_i^n)}{T_i} = \frac{\Delta a_i}{T_i} n a_i^{n-1} = -\frac{FDna^{(n-1/2)}}{\pi m \sqrt{GM}}.$$

We see that if $n=1/2$ than this value will not depend on time. Since the orbit evolution is much longer than the orbital period, we can assume the major semi-axis decrease as continuous process. The value of square root of major semi-axis will decrease by the linear law:

$$\sqrt{a} = \sqrt{a_0} - \frac{FD}{2\pi m \sqrt{GM}} t.$$

Here a_0 is the initial major semi-axis equal to

$$a_0 = R + \frac{h_P + h_A}{2} = 11470 \text{ km}.$$

Here R is the radius of the Earth, h_P and h_A are the altitude values in the perigee and apogee. During the orbit evolution the value of h_P is changing a little. The radius of circular orbit will be equal to

$$a_C = R + h_P = 6570 \text{ km}.$$

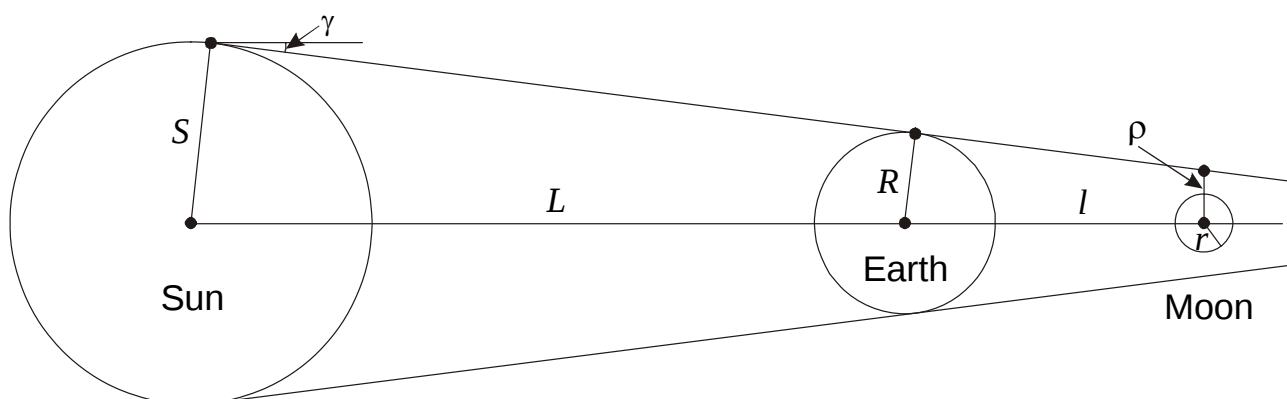
With account of $D=R$, the time of orbit evolution is equal to

$$T_C = \frac{2\pi m \sqrt{GM}}{FD} (\sqrt{a_0} - \sqrt{a_C}) = 1.6 \cdot 10^8 \text{ sec}$$

or about 5 years.

3. Problem. The magnitude of total umbral lunar eclipse is equal to 1.865. Please find the duration of totality. The expansion of the umbra caused by atmosphere can be disregarded. (*O.S. Ugolnikov*)

3. Решение. The value of eclipse magnitude is too high. We have to define the distances between the Sun and the Earth (L) and between the Earth and the Moon (l) for which that value is possible. Let's assume that the eclipse is central and all three bodies are situated along one line. Their position is shown in the figure.

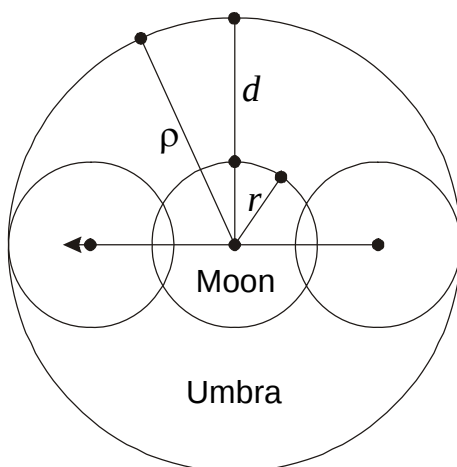


Let's define the umbra radius ρ depending on L and r . The edge of umbra cone tangent to Sun and Earth is inclined to the line "Sun-Earth" by the angle

$$\gamma = \frac{S - R}{L}.$$

This angle is close to the angular radius of the Sun, being quite small (it is equal to 0.26°) and we can use the trigonometric properties of small angle. In particular, we can state that its cosine is equal to unity. The umbra radius is equal to

$$\rho = R - \gamma l = \frac{R(L + l) - Sl}{L}.$$



As we can see in the figure, the central eclipse magnitude is equal to

$$F = 1 + \frac{d}{2r} = 1 + \frac{\rho - r}{2r} = \frac{\rho + r}{2r}.$$

Here r is the radius of the Moon. It is clear that the more the distance between the Sun and the Earth L , and the less the distance between the Sun and the Moon, l , the more the eclipse magnitude. We can see it using two initial formulae of solution. Let's assume that the value of L is maximal (1.017 a.u.), so the Earth reaches the aphelion point of the orbit. If we also assume that the value of l is average (384,400 km) than the eclipse magnitude will not exceed 1.832, that will not met the condition of the problem.

Substituting the minimal value of l (356,400 km) we obtain the maximal magnitude of the lunar eclipse (without account of atmospheric expansion) equal to 1.868. It nearly coincides with the one given in this problem. So, during the eclipse considered here the Moon was near the perigee point of the orbit and crossed the umbra along its diameter. The radius of the umbra is turned out to be equal to 4757 km.

The lunar spatial velocity at the distance l is equal to

$$v = \sqrt{GM \left(\frac{2}{l} - \frac{1}{a} \right)},$$

where a is the major semi-axis of the lunar orbit. Substituting the numbers, we obtain 1.095 km/sec. The umbra also moves relatively the Earth, its velocity is equal to

$$u = v_0 \frac{l}{L},$$

where v_0 is the aphelion orbital velocity of the Earth (equal to 29.3 km/s). The value of u is equal to 0.069 km/s, the direction is the same with the velocity of the Moon v . During the total eclipse the Moon covers the distance

$$D = 2(\rho - r),$$

and the total eclipse duration is equal to

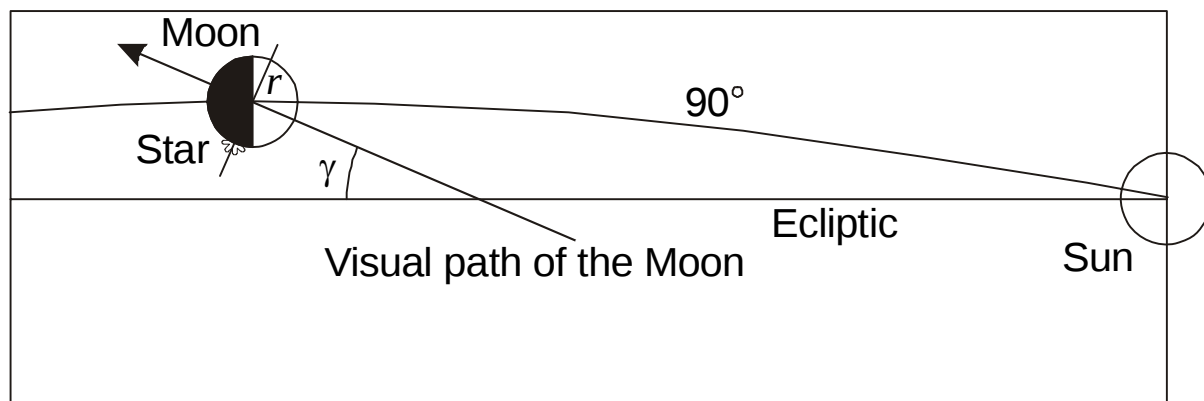
$$T = \frac{D}{v-u} = \frac{2(\rho - r)}{v-u},$$

or 5885 seconds or 1 hour and 38.1 minutes. Here we should notice that it is not the maximal duration of the total lunar eclipse that is reached during the apogee eclipses with smaller magnitude.

Note. In the ephemerides of the lunar eclipses published in astronomical calendars and handbooks you can find the eclipses with magnitudes higher than 1.868. This fact is due to the account of atmospheric expansion of the umbra, which size is considered to be more than the geometrical value.

4. Problem. The grazing occultation of the star by the Moon is observed in the zenith at the Earth's equator. The Moon is exactly in the first quarter. Please find the maximum possible angular distance between the star being occulted and the closest "horn" of the Moon (the crossing point of limb and terminator) in the grazing moment. The orbit of the Moon can be considered to be circular. (O.S. Ugolnikov)

4. Solution. Let's draw the positions of Sun, Moon, star and ecliptic line during the occultation event. The cusps of the Moon are directed along the major circle of the celestial sphere connecting the Sun and the Moon. This circle is shown as the arc in the figure. We have to note that due to the inclination of the lunar orbit this circle does not coincide (in general) with ecliptic and visual path of the Moon on the celestial sphere.



Since the Sun is always situated on the ecliptic and the angular distance between the Sun and the Moon in the first quarter is equal to 90, cusps of the Moon are directed parallel to the ecliptic, not depending on the position of the Moon relatively the ecliptic. If the lunar visual motion relatively the star is directed parallel to the ecliptic, than the grazing occultation will be observed exactly at the "horn" point, the crossing of limb and terminator.

The angular distance between the star and the "horn" depends on the inclination of the lunar path to the ecliptic γ . So we have to define the maximal value of this angle. If we had observed from the center of the Earth, the problem would be much simpler. The value would be equal to the inclination of the lunar orbit, i , equal to 5.15° . It would be reached, when the Moon had crossed the node of its orbit. But really we observe from the surface of rotating Earth and the value of γ can exceed the value of i .

According to the problem, the occultation is observed in the zenith at the equator of the Earth. Since we have to find the maximal value of angle γ , we assume that the Moon is situated on the ecliptic crossing the orbit node. Let it be the ascending node.

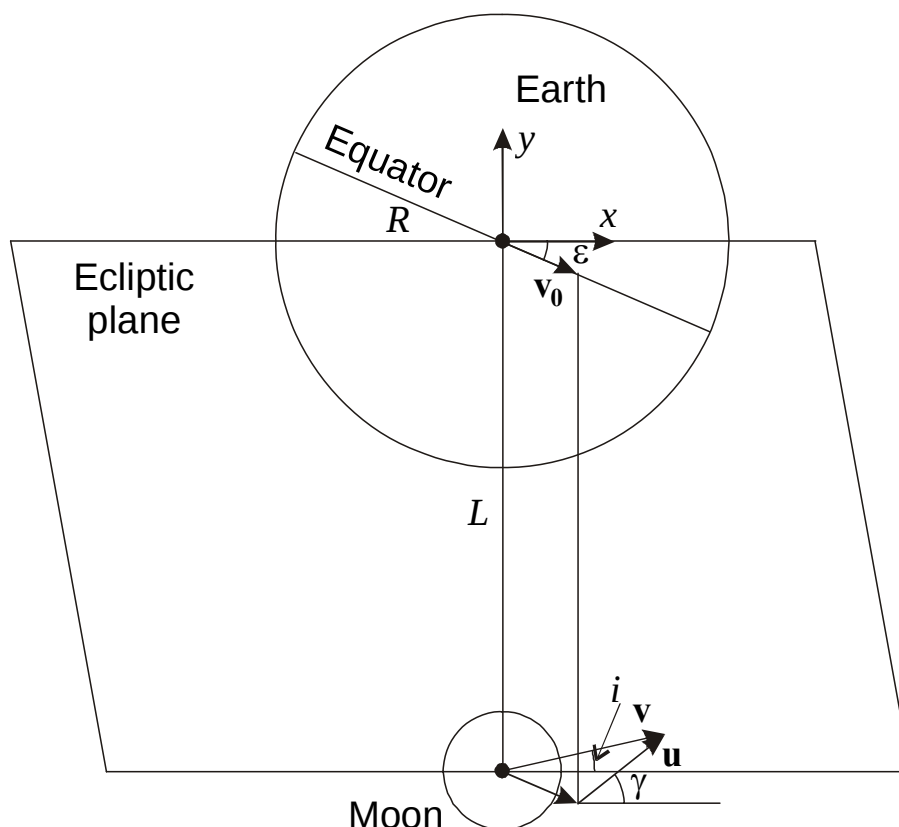


Figure shows the configuration of the Earth and the Moon on the ecliptic plane. The Moon crosses this plane by the angle i and moves northwards. We denote its velocity as v . Considering the orbit as circular, we find this velocity:

$$v = \sqrt{\frac{GM}{L}}.$$

Here M is the mass of the Earth, L – the distance between the Earth and the Moon. The velocity is equal to 1.02 km/sec. The observer is situated on the equator and moves with it by the angle ϵ (23.4°) to the ecliptic. The velocity is equal to

$$v_0 = \frac{2\pi R}{T}.$$

Here R is the radius of the Earth, T is the duration of sidereal day. The value is equal to 0.465 km/s. The velocity of the Moon relatively the observer is the vector difference of two velocities above:

$$\mathbf{u} = \mathbf{v} - \mathbf{v}_0.$$

The angle γ to find is the angle between vector $\mathbf{u}$ and the ecliptic plane. This angle is maximal if the vertical components of vectors $\mathbf{v}$ and $\mathbf{v}_0$ are opposite and the observer moves southwards. It is if the Moon is seen in the vernal equinox point. The situation is the same if the Moon is in descending angle and is seen in the autumn equinox point.

We define the coordinate system (x, y) as shown in the figure and make a projection of vector difference above on the axes of the system:

$$\begin{aligned}u_x &= v \cos i - v_0 \cos \varepsilon, \\u_y &= v \sin i + v_0 \sin \varepsilon.\end{aligned}$$

The angle γ is equal to

$$\gamma = \arctan \frac{u_y}{u_x} = \arctan \frac{v \sin i + v_0 \sin \varepsilon}{v \cos i - v_0 \cos \varepsilon} = 25.1^\circ.$$

We have to note that this angle is almost 5 times larger than for the case of geocentric observations. The angular distance between the star and “horn” is equal to

$$\sigma = 2\rho \sin \frac{\gamma}{2} = 2 \frac{r}{L} \sin \frac{\gamma}{2}.$$

Here ρ is the angular radius of the Moon. Transforming this angle to the degree scale, we obtain 0.113° or $6.8'$. If the grazing occultation occurs at the dark edge of the Moon (as shown in the first figure of solution), that angular distance will be enough to observe the event using the binocular or telescope even in the case of faint star.

5. Problem. The minor planet moves around the Sun in the ecliptic plane, never coming inside the orbit of the Earth. The conditions of its observations exactly repeat in 2 years, and its visible magnitude changes on 8^m with the same period. Please find the minimum possible value of the eccentricity of the asteroid's orbit. The asteroid is the smooth spherical uniform ball with constant surface albedo. Orbit of the Earth can be considered to be circular. (*O.S. Ugolnikov*)

5. Solution. In two years the Earth completes two revolutions around the Sun returning to the same point of the orbit. Since the conditions of asteroid observations are the same, it also returns to the same orbit point in two years. Since it does not come inside the orbit of the Earth, its orbit major semi-axis is not less than 1 a.u. and orbital period is not less than 1 year. The number of completed revolutions in 2 years is one or two.

But if this number was two and orbital period was equal to 1 year, asteroid would come inside the orbit of the Earth (in the case of elliptical orbit) or would be in the fixed position relatively the line “Sun-Earth” not changing the magnitude (in the case of circular orbit). Possible axial rotation of the asteroid does not change the picture, since the asteroid has the uniform surface. Finally, the orbital period of asteroid is equal to 2 years. According to Third Kepler law, the major semi-axis of the orbit is equal to $2^{2/3}$ or 1.587 a.u.

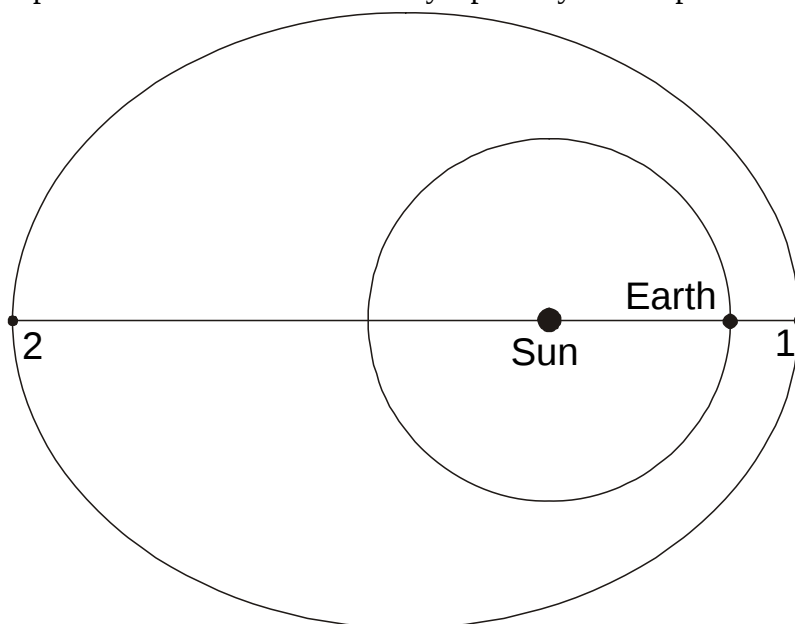
Asteroid is the uniform ball with smooth surface. This case its magnitude does not depend on the phase angle (the angle between the directions from the asteroid to the Sun and the Earth). The magnitude changes are related with the changes of distances between the Sun and asteroid and between the asteroid and the Earth. The asteroid brightness is reverse-proportional to the squares of both distances and the magnitude can be expressed as follows:

$$m = m_0 + 5 \lg d + 5 \lg r.$$

Here d and r are the distances from the asteroid to the Earth and to the Sun expressed in astronomical units, m_0 is the asteroid absolute magnitude (the magnitude for the case $d = r = 1$ a.u.).

Let e be the eccentricity of asteroid orbit. In the case of definite values of e and orbital period (two years) maximal amplitude of brightness changes will be reached in the case shown in the figure. During the opposition asteroid is in the orbit perihelion, position 1 in the figure. The values of d and r reach the minimum simultaneously:

$$\begin{aligned}d_1 &= a(1 - e) - a_0, \\r_1 &= a(1 - e).\end{aligned}$$



Here a_0 is the radius of Earth's orbit. The brightness of the asteroid reaches the maximal possible value. In one year the Earth returns to the same point, but asteroid is in aphelion and in the conjunction with the Sun. The values of d and r reach the maximum simultaneously:

$$\begin{aligned} d_2 &= a(1+e) + a_0, \\ r_2 &= a(1+e). \end{aligned}$$

The brightness of asteroid reaches the absolute minimal value. Since the amplitude increases with the eccentricity, to find the minimal value of e , we have to find for the configuration described above for given amplitude -8^m :

$$5 \lg d_2 + 5 \lg r_2 - 5 \lg d_1 - 5 \lg r_1 = 8,$$

$$\frac{d_2 r_2}{d_1 r_1} = \frac{(a(1+e) + a_0) a(1+e)}{(a(1-e) - a_0) a(1-e)} = K = 10^{1.6} = 39.8.$$

The solution of square equation gives the answer:

$$e = \frac{2(K+1)a - (K-1)a_0 \pm \sqrt{(K-1)^2 a_0^2 + 16Ka^2}}{2(K-1)a}.$$

Just one of two solutions (with the sign « $\rightarrow$ ») has the physical sense ($0 < e < 1$). The eccentricity is equal to 0.284, the perihelion distance is equal to 1.137 a.u. We see that the asteroid really does not come inside the orbit of the Earth. If the eccentricity was less than 0.284, the amplitude (even in the case of opposition in perihelion and conjunction in aphelion) would be also less than 8^m . Thus, we had found the minimal possible value of eccentricity.

6. Problem. The observer on the Earth had measured the angular distance between the stars **X** and **Y**, both situated on the ecliptic, and obtained 30° with exactness $0.1''$. The star **X** is situated westwards from the star **Y**, so it has less ecliptic longitude. During the observation moment both stars were situated westwards from the Sun, the ecliptic longitude difference of the Sun and star **X** was equal to 100° . Please find the angular distance between stars **X** and **Y** after three months. How will this distance change if we observe from the Sun? The parallax values of the stars **X** and **Y** are equal to $0.5''$ and $0.2''$, respectively. Please disregard the eccentricity of the Earth's orbit, self motions of the stars and all atmospheric effects. (E.N. Fadeev)

6. Solution. The coordinates of the stars change by two reasons: light aberration and parallax of the stars. We will analyze these effects separately.

Light aberration is caused by orbital motion of the Earth and the finite value of the speed of light. It moves the star towards the apex of the Earth's motion. The ecliptic longitude of the apex is 90° less than the one of the Sun. For the stars on the ecliptic the change of ecliptic longitude is equal to:

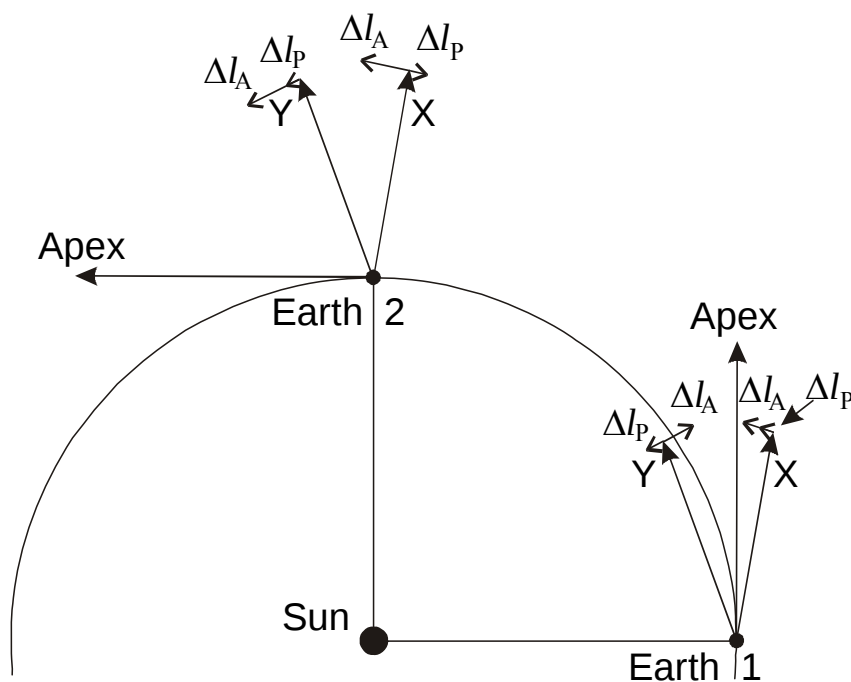
$$\Delta l_A = \frac{v}{c} \sin(a - l) = -k \cdot \cos(l_0 - l).$$

Here v is the velocity of the Earth, c is the light velocity, k is the aberration constant, equal to $20.5''$ in degree scale, a is the apex ecliptic latitude, l_0 is the ecliptic latitude of the Sun, l is the ecliptic latitude of the star.

Parallax shift of the star is always directed towards the Sun. The change of ecliptic longitude is equal to

$$\Delta l_P = \pi \cdot \sin(l_0 - l).$$

Here π is the parallax of the star. Since the both shift values are too small, the common shift is the sum of these values, and we can use the observed values of star ecliptic longitude in these formulae.



According to the problem formulation, in the moment 1:

$$\begin{aligned} l_{01} - l_X &= 100^\circ, \\ l_{01} - l_Y &= 70^\circ. \end{aligned}$$

The values of ecliptic longitude shifts owing to aberration and parallax are equal to:

$$\begin{aligned} \Delta l_{X1} &= -k \cdot \cos(l_{01} - l_X) + \pi_X \cdot \sin(l_{01} - l_X) = +3.6'' + 0.5'' = +4.1'', \\ \Delta l_{Y1} &= -k \cdot \cos(l_{01} - l_Y) + \pi_Y \cdot \sin(l_{01} - l_Y) = -7.0'' + 0.2'' = -6.8''. \end{aligned}$$

The difference of ecliptic longitudes $(l_Y - l_X)_1$ in the moment 1 is equal to 30 and $10.9''$ less than heliocentric difference $(l_Y - l_X)$. Thus, the heliocentric difference of longitudes is equal to $30^\circ 00' 10.9''$. This is the answer for the second question of the problem.

To answer on the first question, we see that in three months

$$\begin{aligned} l_{02} - l_X &= 190^\circ, \\ l_{02} - l_Y &= 160^\circ. \end{aligned}$$

According to this,

$$\begin{aligned} \Delta l_{X2} &= -k \cdot \cos(l_{02} - l_X) + \pi_X \cdot \sin(l_{02} - l_X) = +20.2'' - 0.1'' = +20.1'', \\ \Delta l_{Y2} &= -k \cdot \cos(l_{02} - l_Y) + \pi_Y \cdot \sin(l_{02} - l_Y) = +19.3'' + 0.1'' = +19.4''. \end{aligned}$$

The ecliptic longitude difference $(l_Y - l_X)_2$ is $0.7''$ less than the heliocentric value $(l_Y - l_X)$ and equal to $30^\circ 00' 10.2''$.

7. Problem. In March 1997 we saw the bright comet Hale-Bopp with magnitude -1.5^m . Being observed from Earth, the brightest inner part of the comet's tail had the length about 10° and width about 1° . Imagine that the same time the spaceship with astronauts arrived to the comet and landed on its core at the side opposite relatively the Sun. Will the astronauts see the stars in the sky when they come to the surface of the core? (*O.S. Ugolnikov*)

7. Solution. To answer on the question, we have to look how do the brightness characteristics of expanded objects change when we fly to them or from them. If we approach to comet Hale-Bopp in 2 times for example, it will be brighter in 4 times and will have the angular square 4 times more than before. The surface brightness (or the magnitude of the angular square unit) will not change. If we come inside the expanded object, its emission will be seen from the major part of the sky, but the surface brightness will not be higher (actually it will be lower) than the one observed from large distance.

The surface brightness of the tail of comet Hale-Bopp is equal to -1.5^m per 10 square degrees. Each square degree contains 3600^2 square angular seconds. The magnitude of one square second is equal to

$$m = -1.5 + 2.5 \lg(10 \cdot 3600^2) = 18.8.$$

When we land on the comet surface at the side where the tail is visible, its surface brightness will be the same. But it is just $4\text{--}5^m$ brighter than the moonless night sky on the Earth. The comet night sky will be like evening sky during the nautical twilight. The human eye will see the stars up to $4m$ in these conditions. Finally, the astronauts will see the stars from the surface of the core of comet Hale-Bopp.

8. Problem. The star has the surface temperature 15000 K and the radius equal to 10 radii of the Sun. During the last 100 years this star produces the uniform stellar wind blowing with the velocity 20 km/s. This substance created the shell of gas and dust around the star with optical depth equal to 0.2. Please calculate the radii of inner and outer visible edges of the shell, find the dependence of the dust density in the shell on the distance from the star. Please find the temperature at the outer edge of the shell, mass of the shell and the mass loss rate of the star. The dust particles have the radius equal to 1 μm , density equal to 3 g/cm^3 and fusion temperature equal to 1500 K. Consider that the mass of the gas is 200 times larger than the mass of the dust, but light absorption is caused only by dust. (*A.M. Tatarnikov*)

8. Solution. We know that the mass loss rate $\dot{M}$ is constant. Let's divide the shell into the thin layers with the thickness Δr . Masses of each layer are the same, we denote it as ΔM . The number of particles is also the same for all layers. If the layer radius is R , then the density of the layer will be equal to

$$\rho(R) = \frac{\Delta M}{4\pi R^2 \Delta r} = \rho_{in} \left(\frac{R_{in}}{R} \right)^2.$$

Here R_{in} is the radius of the inner border of the shell and ρ_{in} is the density there. Last formula is the dependency of the density on the distance from the star. To find the temperature dependency, we denote dust particle radius as a . The energy falling to the particle is equal to the energy emitting by the particle to the surrounding space. If we denote the radius and temperature of the star as R_* и T_* , than we will have

$$\frac{4\pi\sigma T_*^4 R_*^2}{4\pi R^2} \cdot \pi a^2 = 4\pi\sigma a^2 T^4.$$

The temperature of particle is equal to

$$T = T_* \sqrt{\frac{R_*}{2R}}.$$

At the inner border of the shell the temperature must be equal to the dust fusion temperature. This case the dust will not reflect the emission closer to the star. Denoting the fusion temperature as T_0 , we obtain

$$R_{in} = \frac{R_*}{2} \left(\frac{T_*}{T} \right)^2.$$

It is equal to 500 solar radii or 50 radii of the star. The outer border radius is more simple to calculate:

$$R_{out} = v \cdot t,$$

Here v is the stellar wind velocity (20 km/sec), t is the time of wind outflow (100 years). The radius is equal to $6.3 \cdot 10^{10}$ km or 90000 solar radii or 9000 radii of the star. We see that outer border radius is many times more than the inner border radius. Physically it shows that the shell does create at the time t . The temperature of the outer border is equal to

$$T_{out} = T_* \sqrt{\frac{R_*}{2R_{out}}} = 110 \text{ K}.$$

To find the mass of the shell, we have to express the optical depth of the shell. We assume that the particles absorb the emission as the black balls with radius a . The probability of the photon to be absorbed in the layer with radius R and thickness ΔR , or the optical depth of the layer, is equal to

$$\Delta\tau(R) = n(R)\Delta R \cdot \pi a^2 = n_{in} \left(\frac{R_{in}}{R} \right)^2 \pi a^2 \Delta R.$$

Here $n(R)$ is the particle concentration at the distance R , n_{in} is their concentration at the inner border of the shell. Total optical depth is the sum of the optical depths of all layers. It is expressed as the integral

$$\tau = \int_{R_{in}}^{R_{out}} d\tau(R) = \int_{R_{in}}^{R_{out}} n_{in} \left(\frac{R_{in}}{R} \right)^2 \pi a^2 dR = \pi a^2 n_{in} R_{in}^2 \left(\frac{1}{R_{in}} - \frac{1}{R_{out}} \right) \approx \pi a^2 n_{in} R_{in}.$$

Here we take into account that the outer radius of the shell is sufficiently larger than the inner radius. The value of optical depth is known, and we can use this formula to calculate the concentration n_{in} . The number of particles in thin layer with radius R and thickness ΔR is equal to

$$\Delta N(R) = n(R) \cdot 4\pi R^2 \Delta R = n_{in} \cdot 4\pi R_{in}^2 \Delta R = \frac{4 R_{in}}{a^2} \Delta R.$$

This value does not depend on radius, since the mass outflow is constant. Taking it into account and knowing that outer radius is many times less than the inner one, we obtain

$$N = \frac{4\tau R_{in} R_{out}}{a^2}.$$

To find the total mass, we remember that the mass of gas is K times more than the mass of dust. If ρ_0 is the dust density, then the shell mass can be expressed as follows:

$$M = \frac{4}{3} \pi a^3 \rho_0 \cdot NK = \frac{16\pi\tau R_{in} R_{out} a \rho_0 K}{3}.$$

It is equal to $4 \cdot 10^{25}$ kg or $2 \cdot 10^{-5}$ of the solar mass. This mass was released in 100 years, so the mass loss rate is equal to $4 \cdot 10^{23}$ kg or $2 \cdot 10^{-7}$ solar masses per year.

9. Problem. The gamma-ray bursts sometimes happen in the distant galaxies. These are the short (about several seconds) bursts of gamma-ray emission with average energy of the photon equal to 1 MeV. To be registered on the Earth, the flux of such photons must be not less than 50 phot/(cm²·s). The luminosity of the burst is equal to 10^{49} ergs per second, this energy is released inside two opposite cones with angle at the top equal to 10° . The gamma-ray bursts are registered on the Earth once a week. What is the frequency of gamma-ray bursts in one definite galaxy? How much times more or less bursts we would see, if the cones of their emission were two times narrower? (*M.E. Prokhorov*)

9. Solution. Let's find which part of the sphere is covered by two cones of gamma-ray emission. These cones draw two circles with radius 5° or 0.087 radians. We denote this angle as ρ . This angle is quite small and we consider the circles as plane figures. The part of the sphere covered by cones is equal to the ratio of circles and sphere squares:

$$b = \frac{2\pi\rho^2}{4\pi} = \frac{\rho^2}{2} = 0.004.$$

The energy of gamma-ray photon is equal to 1 MeV or $1.6 \cdot 10^{-6}$ ergs. Thus, the gamma-ray source emits $6 \cdot 10^{54}$ photons. We denote this value as J_0 . We find the maximal distance to the source to be observed by the device with sensitivity E :

$$E = \frac{J_0}{4\pi b R^2}; \quad R = \frac{1}{2} \sqrt{\frac{J_0}{\pi b E}} = \frac{1}{\rho} \sqrt{\frac{J_0}{2\pi E}}.$$

Here we assume that the Earth is situated inside one of two emission cones. Thus, we can detect the gamma-ray burst at the distance up to 500 Mpc. The change of flux due to the Universe expansion is not sufficient at this distance.

The average concentration of galaxies in the Universe is equal to 0.01 Mpc^{-3} . We can obtain this value from the total number of galaxies in the Universe (10^{10}) and the size of the Universe (10 Gpc). The number of galaxies inside the sphere with radius R is equal to

$$N = \frac{4}{3} \pi R^3 n = \frac{n}{6\sqrt{\pi}} \left(\frac{J_0}{bE} \right)^{3/2} \sim 5 \cdot 10^6.$$

We see the bursts with the frequency F equal to 1 burst per week or 50 bursts per year in this number of galaxies. But we cannot see all the bursts in these galaxies as we cannot always be inside the emission cones. Total frequency of burst in these galaxies is equal to

$$F_0 = \frac{F}{b}$$

or 12500 bursts per year. The bursts frequency in one galaxy is equal to

$$F_1 = \frac{F_0}{N} = F \frac{6\sqrt{\pi b}}{n} \left(\frac{E}{J_0} \right)^{3/2} = F \frac{3\sqrt{2\pi}}{n} \left(\frac{E}{J_0} \right)^{3/2} \rho.$$

or once in 400 years. We will see 1/250 of these bursts, so the frequency of observable bursts in one galaxy is once in 100,000 years.

We see that this value in the formula is proportional to the square root of b or proportional to the cone angular radius ρ . If the radius was two times less, the total frequency of the bursts in one galaxy would be two times less. But the value of F_0 is reverse proportional to the square root of b , so the total number of observable bursts would increase in two times!

It is easy to explain. If the cone radius was two times less, we could see two times more distant gamma-ray bursts. They would fulfill 8 times larger volume in the Universe. The probability to be inside the cones would be 4 times less. So, the frequency of gamma-ray burst observations would increase in two times.

10. Problem. We know that the temperature of Cosmic Microwave Background in the direction with Galactic coordinates $l = 264^\circ$ and $b = 48^\circ$ is maximal, being by $\Delta T = 3.35$ mK more than average value. Please find the velocity of our Galaxy as a whole relatively the Cosmic Microwave Background. (*E.N. Fadeev*)

10. Solution. Firstly we have to find the velocity of Sun relatively the Cosmic Microwave Background (CMB). The temperature change is related with the Doppler effect:

$$\frac{\lambda_0 - \lambda}{\lambda} = \frac{v}{c}.$$

Here λ_0 is the average (through the sky) wavelength of the CMB maximum, λ is the one for the direction described above, v is the velocity of the Sun relatively CMB. Since the CMB emission is thermal, the wavelength of maximal emission is reverse proportional to the temperature:

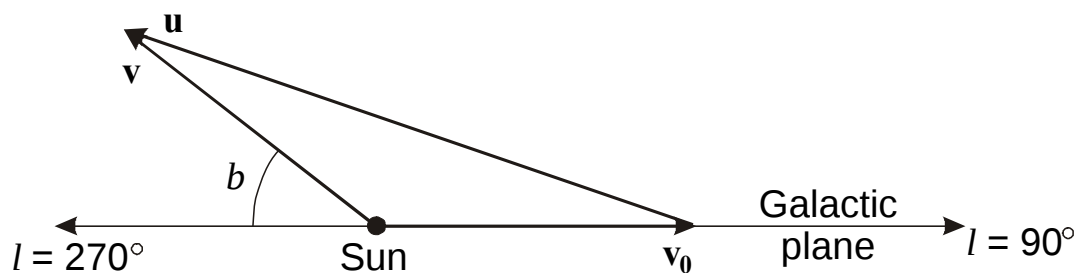
$$\lambda \text{ (cm)} = \frac{0.29}{T}$$

The velocity is equal to

$$v = c \cdot \frac{T - T_0}{T} = c \frac{\Delta T}{T}.$$

Thus, the Sun moves relatively CMB with the velocity 368 km/sec in the direction with galactic coordinates $l = 264^\circ$ and $b = 48^\circ$. But the Sun moves relatively the center of the Galaxy with the velocity v_0 equal to 220 km/sec and directed to the point with galactic coordinates $l_0 = 90^\circ$ and $b_0 = 0^\circ$. The vector of common Galaxy velocity relatively CMB is equal to

$$\mathbf{u} = \mathbf{v} - \mathbf{v}_0.$$



We see that the galactic longitude l corresponding the vector $\mathbf{v}$ is close to 270° , and we can assume that all three vectors are in the figure plane perpendicular to the galactic plane. This case the value of the velocity u can be calculated as follows:

$$u = \sqrt{v^2 + v_0^2 - 2vv_0 \cos(180^\circ - b)}.$$

It is equal to 540 km/sec. If we take into account the difference of l and 270° , the answer will be the same with the exactness of 1 km/sec.