Romanian National Astronomy and Astrophysics Olympiad 2014

"Theoretical and Data Analysis Problems with Solutions"

Romania Astronomy and Astrophysics Olympiad, 2014
Theoretical Problems and Solutions

Compiled By
Science Olympiad Blog

National Astronomy and Astrophysics Olympiad 2014

Seniors

Theoretic Phase

PART I - Short Problems

Problem 1

The height of the sky! The ancient Greeks knew that the diameter of the Earth is small compared to the distance to the stars. For example, there is a legend that the god Hephaestus accidentally dropped his anvil on Earth. It took τ=9 days for the anvil to eventually hit the ground.
Estimate the "height of the sky", in accordance to the beliefs of the ancient Greeks. It is known that the rotation of the Moon around the Earth is T_L=27,3 days and the radius of the Moon is a_L= 384400 km.

Problem 2

The density of exoplanet X. A radio source is situated on the surface of exoplanet X's satellite. The source constantly emits radio waves but an observer from Earth can't always record the emitted signals because the satellite is occasionally eclipsed by the planet.
Making use of a graphic which illustrates the frequency of the registered signal relative to the time, *find* the density of the exoplanet. The orbit of the satellite is circular and the observer is situated within the same plane as the satellite's orbit. The following data is known: the radius of the exoplanet, *R*; the universal gravitational constant, *K*; the speed of light in vacuum, *c*. It is also known that the satellite revolves very close to the surface of the exoplanet.

Problem 3

Angular height of a star. The radar of an astronomical observatory is installed on a rocky plateau near the seashore, at the height *h* which is above the sea level. The receiver of the observatory records only the electromagnetic signals coming from the star σ. The vector *E* (the intensity of the electric field of these signals) oscillates parallel with the plane and horizontal surface of the sea, independent from the propagation direction of the electromagnetic wave. The intensity of any recorded signal is proportional with E^2. When the indicator of wavelengths show the value λ, the radar receiver records minimum and maximum values.

Determine the angular heights of the the star σ above sea level, at which the radar receiver records electromagnetic signals of maximum and minimum intensities.

Problem 4

Visible star? The Sun parallax is p_{sun} = 8,8” and the parallax of a star, σ, which has the same absolute brightness (luminosity) as the Sun, is p_{star}=0,022”.
Reveal whether this particular star can be observed on the night sky with the naked eye. The following data is known: the distance between Earth and the Sun, r_{ES} = 149000000 km; Earth radius, R_E= 6380 km.

Problem 5

Star Wars. In “Star Wars”, a star with the apparent magnitude of $m_{initial}=3^m$ was cut into four identical smaller stars that had the same density and temperature like the initial one.
Determine the magnitude of the resulting quadruple star and compare it to the magnitude of the initial star.

PART II - Long Problems

Problem 1

Neutronic star. It is well known that many stars form binary systems. One type of binary system consists of a normal star (with the mass m_0 and radius R) and a neutronic star (much more compact and with a larger mass), which revolve arund their own centre of mass. In the following problem the Earth’s movement is neglected.

Based on terrestrial observations of this type of binary system, the following information is known:
- the maximum angular displacement of the normal star is $\Delta\theta$ and the maximum angular displacement of the neutronic star is $\Delta\phi$, as indicated in Figure 1;
- the necessary time for this kind of maximum displacements is τ;
- the radiation attributes of the normal star show that its surface temperature is T and the incident radiant energy per area unit of the Earth’s surface, per time unit, is P;

- the Calcium (Ca) spectrum line of this radiation has a wavelength which differs from the normal one (λ_0) by $\Delta\lambda$, due only to the normal star's gravitational field.

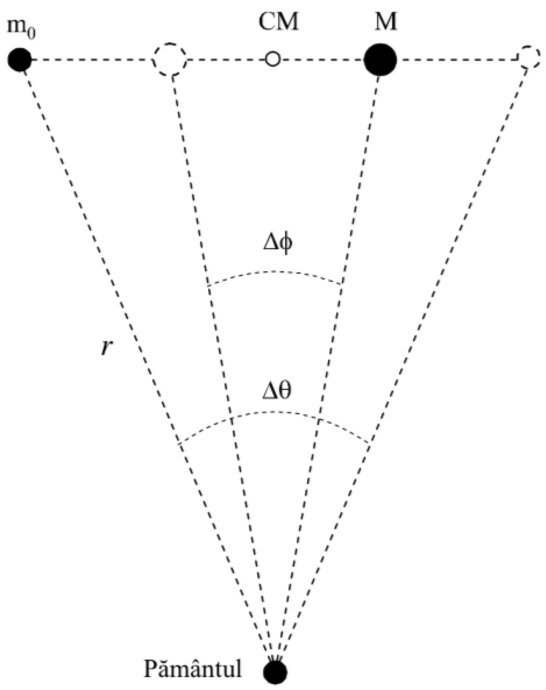

Fig. 1

a) *Find* the distance r between Earth and the binary system presented above by using only the values of the observed units and the universal physical constants involved.

b) Now, let's suppose that $M \gg m_0$, so that the normal star revolves around the neutronic star on a circular orbit with the radius r_0. The normal star starts to emit gas twards the neutronic one with a relative speed of v_0 (relative to the normal star) as indicated in Figure 2. Admitting that the neutronic star is the dominant source of the gravitational action and neglecting the orbit changes of the normal star, you are asked to *determine* the minimum distance r_{min} at which the gas gets close to the neutronic star. It is known that the universal gravitational constant is K.

c) *Determine* the maximum distance r_{max} at which the gas reaches close to the neutronic star.

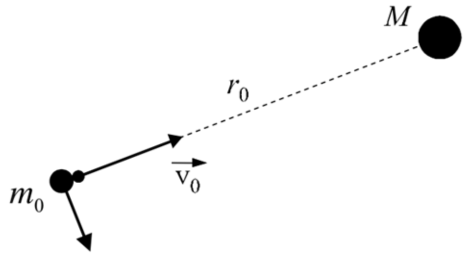

Fig. 2

Problem 2

A. Sun dusk. The dusk and dawn are two events of lengths that depend solely on the place and time of the observation.

a) *Determine* the duration of the dusk/dawn for an observer situated in a place with the latitude φ on equinox days;
b) *Localise* the observer so that during the equinox days, the duration of the dusk/dawn is maximum/minimum;
c) *Determine* the duration of the dusk/dawn for an observer situated in a place with the latitude φ on solstice days;
d) *Localise* the observer so that during the solstice days, the duration of the dusk/dawn is maximum/minimum.

The following data is known: the apparent angular diameter of the Sun, θ = 31'59,3"; the rotation period of the Earth around the Sun T_E=24h, the angle between the equator plane and the ecliptic plane ε=23°27". The effects of atmospheric refraction are neglected.

B. The third cosmic speed. You are asked to *determine* the approximate minimum value of escape velocity that a body must have so that when launched from Earth, it would escape the Solar System forever (third cosmic speed).

The following data is known: $V_0 \approx 30$km/s, the speed of Earth around its circular orbit around the Sun; $v_0 \approx 7{,}9$km/s, the speed of a low orbit satellite that revolves around the Earth (first cosmic speed).

It is also known that $\frac{M_T}{R_T} \ll \frac{M_S}{R_{TS}}$. The body's kinetic energy relative to the Sun is neglected from the moment of launch until reaching the limit of the Earth's gravitational field.

C. Fall from the Earth on the Sun! You are asked to *determine* the minimum speed that is needed for a spaceship to escape Earth's gravity and fall on the Sun's surface. The following data is known: the distance between Earth and the Sun, r_{ES}= $1{,}5 \cdot 10^{11}$ m; the rotation period of the Earth around the Sun, $T_E = 3{,}15 \cdot 10^7$s.

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Romania Astronomy and Astrophysics Olympiad, 2014

Data Analysis Problems and Solutions

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Science Olympiad Blog

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Seniors

Data analysis Phase

Problem 1

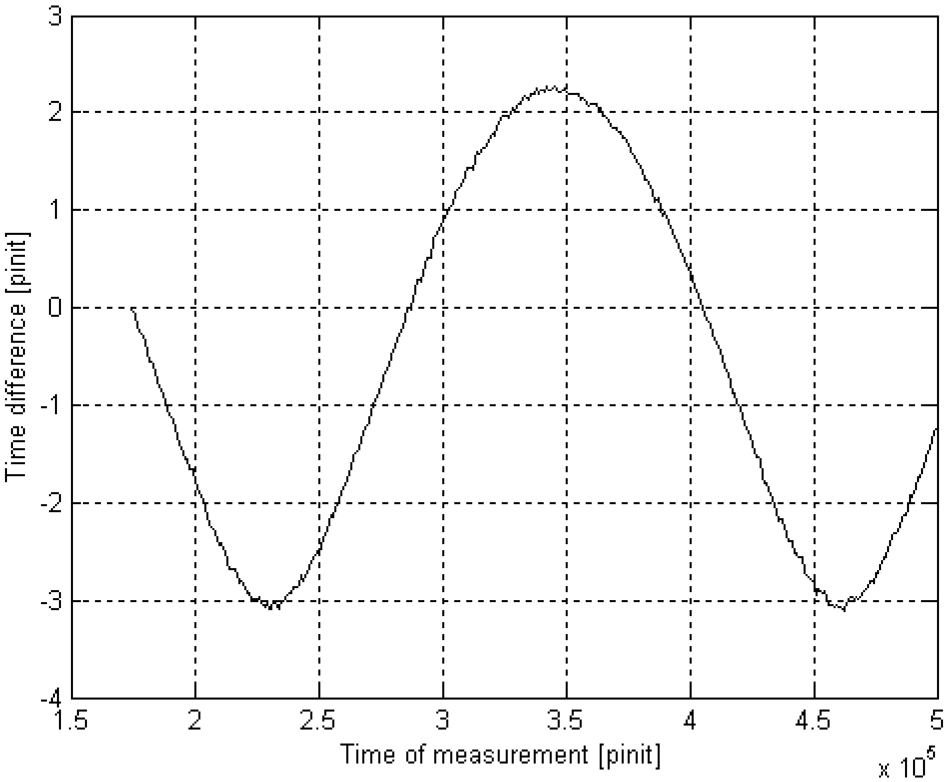
Speed of light. Let's imagine that in a distant future, the Solar System will be occupied by our descendants. A small mining robot installed on the SALTIS asteroid is supervised by Celesta Spacedigger, who happens to also be a passionate amateur astronomer. During the long nights of Saltis, Celesta (character from the Greek mythology) observes the stars and planets, particularly the beautiful planet Saturn. An old but trustworthy astronomical almanac helps her follow certain celestial events such as Titan's eclipses due to Saturn's movement. To her astonishment, Celesta discovers large differences between the time values she noticed while observing Titan's eclipses and the existent values from the almanac. After years of careful observation (as she was detached to stay on SALTIS for a long time), Celesta eventually finds an explanation. The differences are the largest when Saturn is close to the opposition or conjunction with the Sun, both seen from Saltis. Celesta figures out that this is because the speed of light is finite. Also, she discovers that a sketch from the almanac confirms the fact that the synchronizations from its tables are heliocentric (relative to the Sun and not to Saltis). Very satisfied with her discovery, Celesta used these observations to calculate the speed of light.
In the following problem you have to *repeat the computation* done by Celesta using her observations. The units of time and length used by Celesta are fairly different than the ones we use. The unit of time which is called *pinit* is defined so that Saltis's synodic period of rotation is $T_{synodic\ Saltis}$ = 1000 pinit. The unit of length called *seter* is defined so that 1 seter is equal to 10^{-9} the mean distance from the Sun to Saltis. In other words, $r_{Saltis\text{-}Sun}=10^9$ seter.

a) Six (6) records made by Celesta on Titan's eclipses when Saturn was close to the opposition or conjunction are represented below. The columns are described as follows:
   I) the values from the almanac table regarding the moment when an observer situated on the Sun could see the beginning of the eclipse;
   II) the values of Celesta's observations regarding the beginning of the eclipse as seen from Saltis. The accuracy of the synchronizations is ε= 0,03 pinit;
   III) Saturn's position during Titan's eclipse (close to the opposition or conjunction).

Observation number	Almanac table (pinit) I	Observations made by Celesta II	Statement III
1	456,47	450,32	Opposition
2	18,50	12,28	Opposition
3	821,41	815,29	Opposition
4	444,70	450,85	Conjunction
5	615,43	621,52	Conjunction
6	791,94	798,02	Conjunction

Carefully analyzing the data from the table, *estimate* the speed of light expressed in seter/pinit and *state* what is the possible error of the estimation.

b) During those days on Saltis when she feels lonely, Celesta likes to listen to the radio signals coming from Earth. Now that she knows the speed of light, Celesta wants to determine Earth's radius expressed in seter. Her watch is accurately synchronized with the signals coming from Earth. The results from the measurements are shown in the graphic below:



Estimate the radius of planet Earth expressed in meter using Celesta's data from the chart above.

c) Knowing that: 1 au=149,6·10^6 km; c=2,998·10^8 m/s, *determine* the equivalent in meters for 1 seter; the equivalent in seconds for 1 pinit.

d) Estimate the sidereal orbital period of Saltis expressed in years using c) and the graphic above.

Problem 2

The orbit of planets. It is accepted that Earth's orbit around the Sun is a circle with the radius r_E=1 au. In the following table the maximum Eastern and Western angular elongations of Mars and Venus are specified.

a) Admitting that the orbits of the three planets (Mercury, Venus and Earth) relative to the Sun are described as concentric and coplanar circles, let's suppose that Venus/Mercury are in the corresponding position for their maximum Eastern elongation. *Determine* after what time Venus/Mercury will be:
   I) for the first time, in the corresponding position for the maximum Western elongation;
   II) again in the corresponding position for their maximum Eastern elongation.

b) According to the data from the table and using a piece of paper on which Earth's circular orbit around the Sun (which is in the middle) is represented, *localise* Mercury and Venus's approximate position around he Sun and determine the mean distance between Mercury and the Sun and the mean distance between Venus and the Sun.
c) Determine the exact values of Mercury and Venus's orbit parameters, *(a; b; e)*, expressed in au, knowing that:

$$18°\leq\alpha_{\max,\mathrm{East}}^{\mathrm{Mercury}}\leq 28°;\ 18°\leq\alpha_{\max,\mathrm{West}}^{\mathrm{Mercury}}\leq 28°;$$
$$45°\leq\alpha_{\max,\mathrm{East}}^{\mathrm{Venus}}\leq 48°;\ 45°\leq\alpha_{\max,\mathrm{West}}^{\mathrm{Venus}}\leq 48°.$$

Table - The maximum elongations of Mercur and Venus

Mercury: 1989 - 1990				Venus: 1983 - 1990		
Day	$\alpha_{\max,\mathrm{Est}}$	$\alpha_{\max,\mathrm{Vest}}$		Day	$\alpha_{\max,\mathrm{Est}}$	$\alpha_{\max,\mathrm{Vest}}$
8 January 1989	19°			15 June 1983	45°	
18 February 1989		26°		4 November 1983		47°
30 April 1989	21°			21 January 1985	47°	
18 June 1989		23°		12 June 1985		46°
28 August 1989	20°			26 August 1986	46°	
10 October 1989		18°		15 January 1987		47°
22 December 1989	20°			2 April 1988	46°	
1 February 1990		25°		22 August 1988		46°
13 April 1990	20°			8 November 1989	47°	
31 May 1990		25°		30 March 1990		46°
11 August 1990	27°					
24 September 1990		18°				
5 December 1990	21°					

Problem 3

The Great Opposition. On the 28th August 2003, at 17:56 UT (Universal Time), the latest Great Opposition of Mars took place, as the Earth passed between Mars and the Sun, aligning with them. The distance between Earth and Mars had the smallest possible value. The next Great Opposition of Mars will take place in 2018. Someone doesn't understand the special aspect of the Great Opposition and thinks that a Simple Opposition (Earth passing between Mars and the Sun and aligning with them) will happen in 2018, not a Great Opposition.

a) *Determine* the circular orbit parameters of the hypothetical planet *Mars 2* which will be in a Simple Opposition to the Sun as seen from Earth in 2018. Earth's orbit around the Sun is circular.

b) *Estimate* the apparent magnitude of *Mars 2* during its opposition in 2018, as seen from Earth. It is known that the apparent visual magnitude of Mars during the Great Opposition in 2003 was $m_{M,2003} = -2^m$. The physical characteristics of *Mars 2* are identical to those of Mars.

c) The following table lists all the Oppositions of Mars from 1955 to 2037. *Identify* the Oppositions taking place near the Perihelion, the Oppositions near the Aphelion and the Great Oppositions. *State* the criteria used for identification.

Mars's Oppositions, 1955 - 2037		
Opposition date	**Date of maximum proximity**	**Minimum distance between Earth and Mars (ua/ millions of miles)**
12th February 1995	11th February 1995	0,67569/62,8
17th March 1997	20th March 1997	0,65938/61,3
24th April 1999	1st May 1999	0,57846/53,8
13th June 2001	21st June 2001	0,45017/41,8
28th August 2003	27th August 2003	0,37272/34,6
7th November 2005	30th October 2005	0,46406/43,1
24th December 2007	18th December 2007	0,58935/54,8
29th January 2010	27th January 2010	0,66398/61,7
3rd March 2012	5th March 2012	0,67368/62,6
8th April 2014	14th April 2014	0,61756/57,4
22nd May 2016	30th May 2016	0,50321/46,8
27th July 2018	31st July 2018	0,38496/35,8
13th October 2020	6th October 2020	0,41492/38,6
8th December 2022	1st December 2022	0,54447/50,6
16th January 2025	12th January 2025	0,64228/59,7
19th February 2027	20th February 2027	0,67792/63,0
25th March 2029	29th March 2029	0,64722/60,2
4th May 2031	12th May 2031	0,55336/51,4
27th June 2033	5th July 2033	0,42302/39,3
15th September 2035	11th September 2035	0,38041/35,4
19th November 2037	11th November 2037	0,49358/45,9

d) Determine the ratio of the diurnal equatorial horizontal parallax of the Sun (p_{eS}) and the diurnal equatorial horizontal parallax of Mars (p_{eM}).

Known values:
Mars' orbital period: T_M = 780 days;
Earth's orbital period: T_E = 365 days;
The numerical eccentricity of Mars's orbit: e = 0.093.

Problem 4

A. The star Altair and the Sun. For the star Altair (the alpha star from the constellation Aquila) the following data is known: the annual parallax p_0=0,198"; its own movement, μ=0,668"/year; the radial velocity, v_r=-26km/s; the apparent visual magnitude, $m_0=0{,}89^m$.
a) *Determine*: the time span after which the distance between Altair and Sun will be minimum, τ; the minimum distance between Altair and Sun, r_{min}; the apparent magnitude of Altair, *m,* when the distance between the star and Sun is minimum.
The distance between Earth and Sun is known to be *d* = 150000000 km.

B. The heating of the water from the basin.
b) *Estimate* the water temperature rise from an average basin with the dimensions *50 x 20 x 2* m, if the basin could collect the entire energy that astronomers receive from the stars by observing them during the night on optic telescopes in order to obtain information about the structure of the Universe.
The following data is known: for the Sun, according to the apparent magnitude, $m_s=-26{,}8^m$, the solar irradiation constant, k_{Sun}=1,37 kW/m^2; the specific heat of the water, c = 4200 J/(kg·K); the area of the working surface of all the professional telescopes, *S* = 1000 m^2 ; the total work time, t=$3\cdot10^8$ seconds; the magnitude of the objects observed with the telescope, $m\approx1^m$; the energetic efficiency of the used devices, η=10%.

C. The apparent magnitude of the Sun. Admitting that the radiation of the stars is the radiation of black bodies, using the information from the following table *determine* the apparent visual magnitude of the Sun.

Nr. crt.	Name of star	Angular diameter of star (δ_{star})	Star's absolute surface temperature (T_{star})	Star's apparent magnditude (m_{star})	Sun's apparent magnditude (m_{Sun})
1	Betelgeuse	0",047	3200 K	$0{,}4^m$	?
2	Aldebaran	0",020	3800 K	$0{,}9^m$	?
3	Arcturus	0",020	4300 K	$-0{,}04^m$	?
4	Herculis	0",030	2650 K	$3{,}15^m$	?
5	Ceti Mira	0",056	2400 K	$3{,}04^m$	?
6	Antares	0",040	3100 K	$0{,}88^m$	?
7	The Sun	32'	6000 K	-	-

ONAA 2014
SENIORI
Analiza Datelor - BARAJ

Problema 1

a)

$$c = \frac{r_{\text{Saltis-Soare}}}{\Delta t_{\text{Opozitie}}};$$

$$c = \frac{10^9 \text{ seter}}{6{,}22 \text{ pinit}} = \frac{10}{6{,}22} \cdot 10^8 \frac{\text{seter}}{\text{pinit}} \approx 1{,}60 \cdot 10^8 \frac{\text{seter}}{\text{pinit}};$$

$$c = (1{,}60 \pm 0{,}03) \cdot 10^8 \frac{\text{seter}}{\text{pinit}},$$ **3p**

reprezentând viteza luminii, determinată de Celesta, exprimată în seter/pinit.

b) Utilizând desenele din figurile alăturate, rezultă:

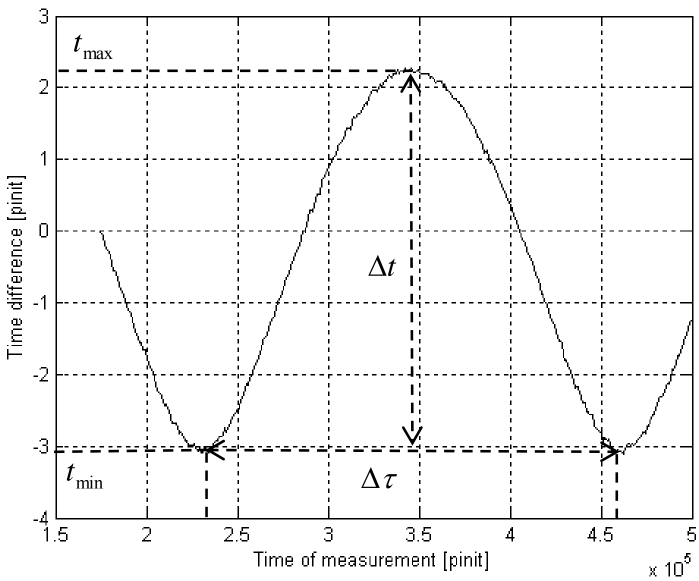


Fig.

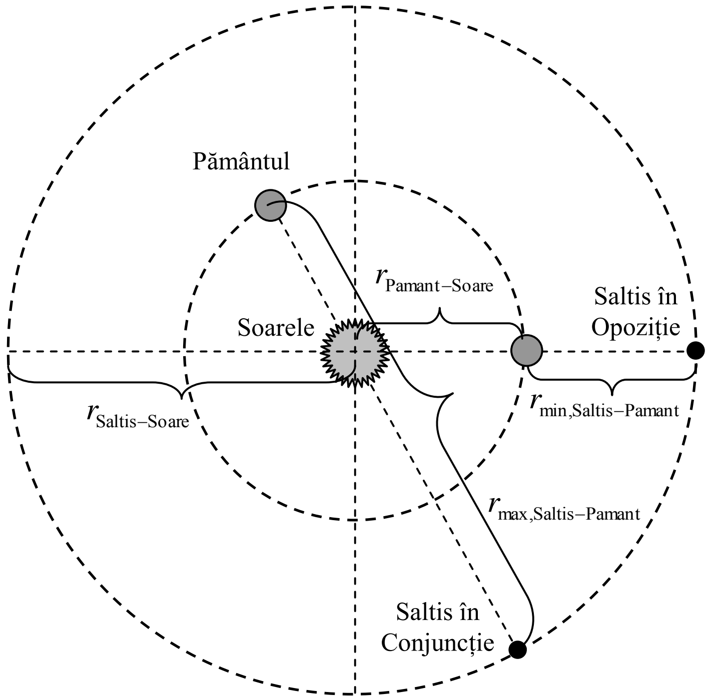


Fig.

$$\Delta t = t_{\max} - t_{\min} \approx 5{,}3\ \text{pinit};$$

$$r_{\text{max,Saltis-Pamant}} - r_{\text{min,Saltis-Pamant}} = 2 \cdot r_{\text{Pamant-Soare}};$$

$$c = 1{,}60 \cdot 10^8\ \frac{\text{seter}}{\text{pinit}};$$

$$\Delta t \cdot c = 5{,}3\ \text{pinit} \cdot 1{,}6 \cdot 10^8\ \frac{\text{seter}}{\text{pinit}} \approx 8{,}5 \cdot 10^8\ \text{seter};$$

$$r_{\text{Pamant-Soare}} = \frac{1}{2} \cdot \Delta t \cdot c = 4{,}25 \cdot 10^8\ \text{seter} = 1\ \text{UA}$$ **2p**

c)

$1\,\text{seter} = 352\ \text{m}; \quad 1\,\text{pinit} \approx 188\ \text{s}$...**2p**

d)

$$T_{\text{sideral,Saltis}} = T_{\text{sideral,Pamant}} \cdot \left(\frac{a_{\text{Saltis}}}{a_{\text{Pamant}}}\right)^{3/2} = 1\,\text{an} \cdot \left(\frac{10^9 \cdot 352\ \text{m}}{149 \cdot 10^9\ \text{m}}\right)^{3/2};$$

$$T_{\text{sideral,Saltis}} = 1\,\text{an} \cdot \left(\frac{352}{149}\right)^{3/2} = (2{,}36)^{3/2}\,\text{ani} \approx 3{,}6\ \text{ani}$$ **3p**

Problema 2

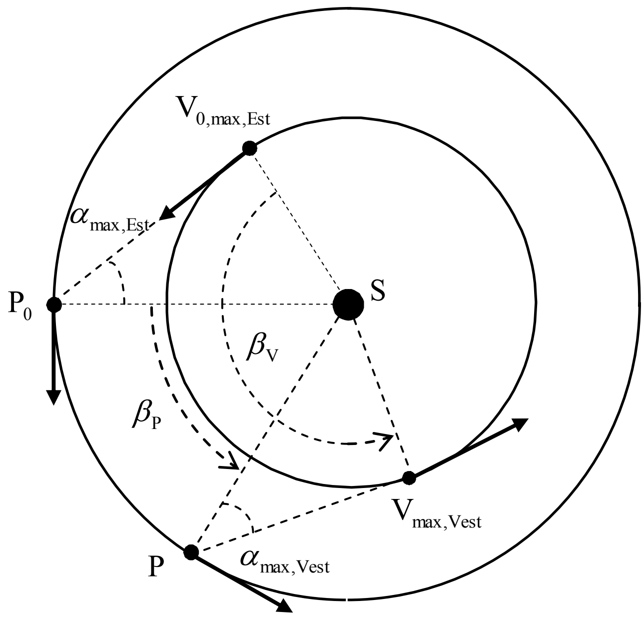
Rezolvare

a)

$$\alpha_{\max,\text{Est,Mercur}} = \alpha_{\max,\text{Vest,Mercur}} = 27^\circ;$$
$$\alpha_{\max,\text{Est,Venus}} = \alpha_{\max,\text{Vest,Venus}} = 47^\circ.$$

1) Pentru planeta Venus:

$$t = \frac{86^\circ}{360^\circ}\cdot\frac{T_\text{P}T_\text{V}}{T_\text{P}-T_\text{V}} \approx 0{,}24\cdot\frac{T_\text{P}T_\text{V}}{T_\text{P}-T_\text{V}} = 0{,}24\cdot\frac{365{,}2\text{ zile}\cdot 234{,}7\text{ zile}}{365{,}2\text{ zile}-234{,}7\text{ zile}} \approx 157{,}6\text{ zile};$$ **1p**

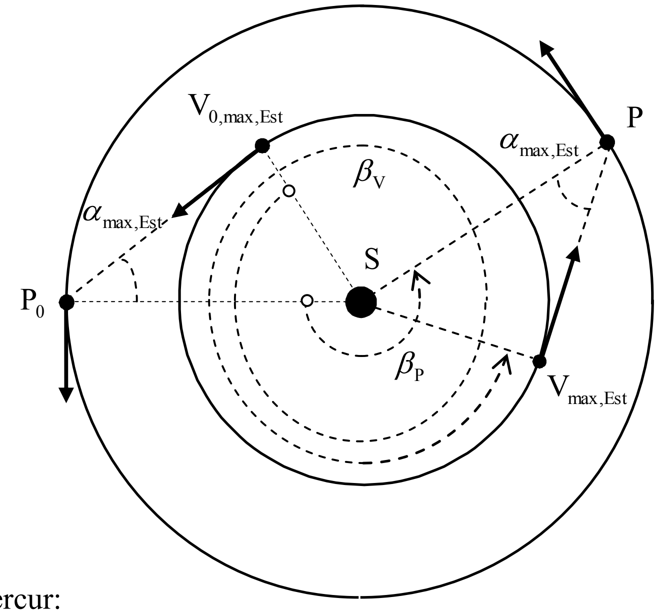


Pentru planeta Mercur:

$$t = \frac{126^\circ}{360^\circ}\cdot\frac{T_\text{P}T_\text{M}}{T_\text{P}-T_\text{M}} \approx 0{,}35\cdot\frac{T_\text{P}T_\text{M}}{T_\text{P}-T_\text{M}} = 0{,}35\cdot\frac{365{,}2\text{ zile}\cdot 88\text{ zile}}{365{,}2\text{ zile}-88\text{ zile}} \approx 40{,}6\text{ zile};$$ **1p**

2) Pentru planeta Venus:

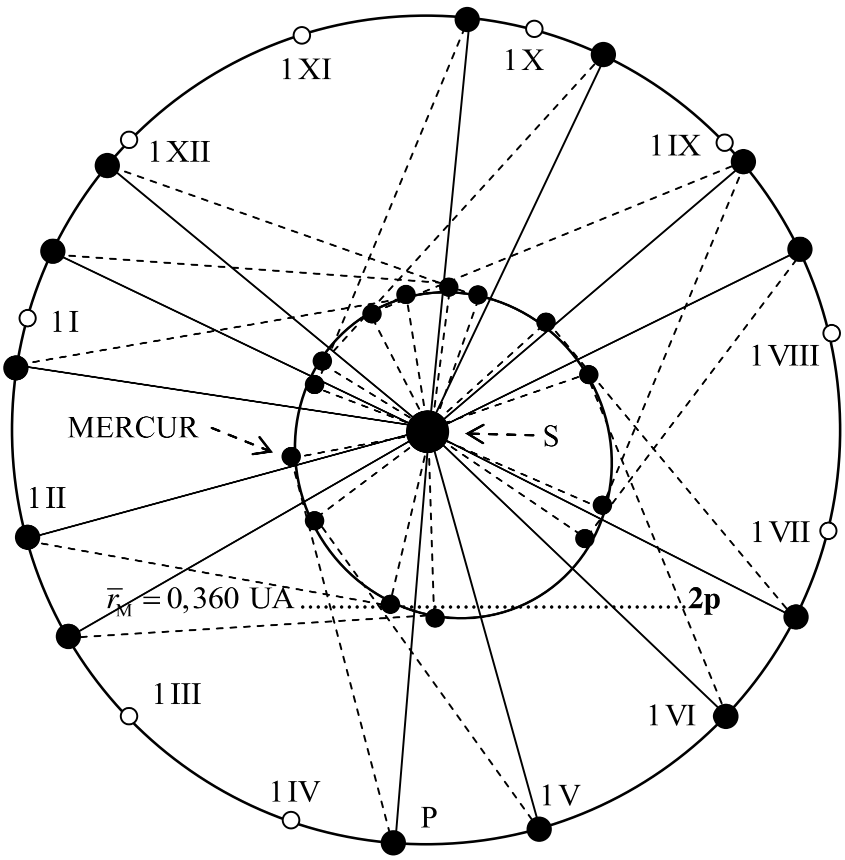
$$t = \frac{T_\text{P}T_\text{V}}{T_\text{P}-T_\text{V}} = \frac{365{,}2\text{ zile}\cdot 234{,}7\text{ zile}}{365{,}2\text{ zile}-234{,}7\text{ zile}} \approx 656{,}8\text{ zile};$$ **1p**



Pentru planeta Mercur:

$$t = \frac{T_{P}T_{M}}{T_{P}-T_{M}} = \frac{365{,}2\ \text{zile}\cdot 88\ \text{zile}}{365{,}2\ \text{zile} - 88\ \text{zile}} \approx 116\ \text{zile}; \ldots\ldots\ldots\ldots\ldots\mathbf{1p}$$

b)



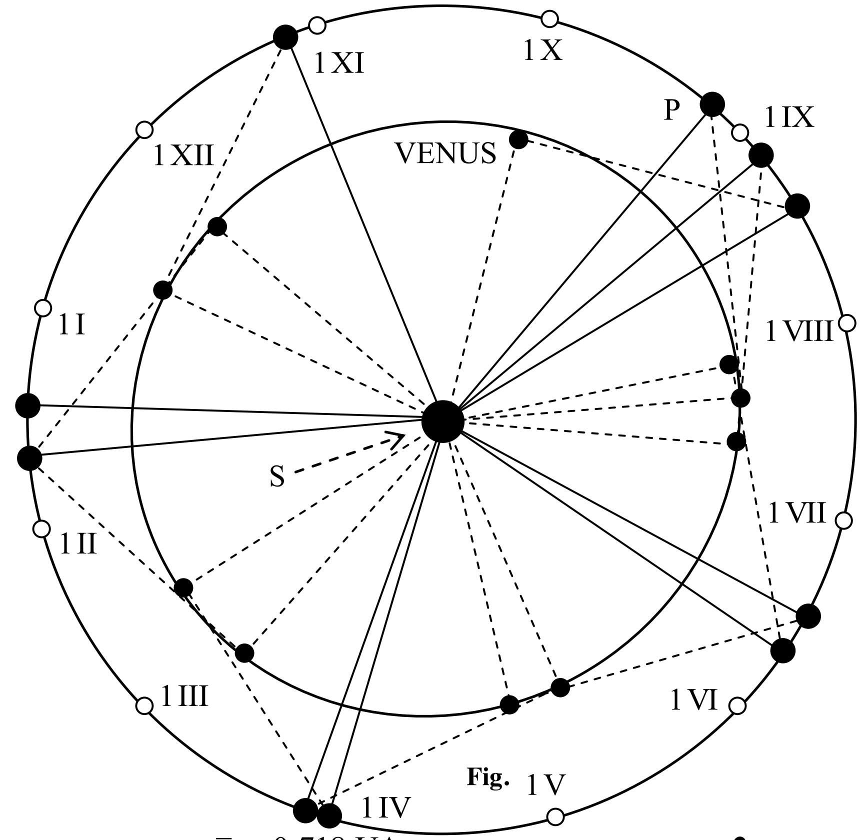


Fig.

$\overline{r}_{\mathrm{V}} = 0,718$ UA **2p**

c)

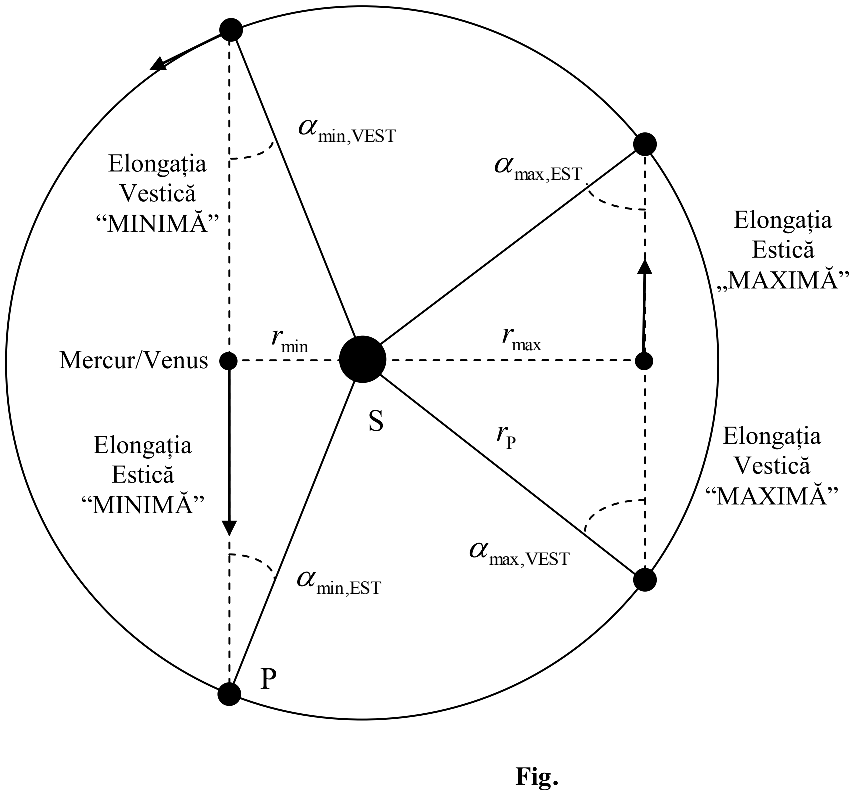


Fig.

- pentru planeta Mercur:

$$a_{\text{Mercur}} = \frac{r_{\text{min,Mercur}} + r_{\text{max,Mercur}}}{2} = 0{,}389\ \text{UA};$$

$$e_{\text{Mercur}} = \frac{r_{\text{max,Mercur}} - r_{\text{min,Mercur}}}{r_{\text{min,Mercur}} + r_{\text{max,Mercur}}} \approx 0{,}020;$$

$$e_{\text{Mercur}} = \sqrt{1 - \frac{b_{\text{Mercur}}^2}{a_{\text{Mercur}}^2}};$$

$$b_{\text{Mercur}} = a_{\text{Mercur}}\sqrt{1 - e_{\text{Mercur}}^2} \approx 0{,}388\ \text{UA};$$ **1p**

- pentru planeta Venus:

$$a_{\text{Venus}} = \frac{r_{\text{min,Venus}} + r_{\text{max,Venus}}}{2} = 0{,}725\ \text{UA};$$

$$e_{\text{Venus}} = \frac{r_{\text{max,Venus}} - r_{\text{min,Venus}}}{r_{\text{min,Venus}} + r_{\text{max,Venus}}} \approx 0{,}024;$$

$$e_{\text{Venus}} = \sqrt{1 - \frac{b_{\text{Venus}}^2}{a_{\text{Venus}}^2}};$$

$$b_{\text{Venus}} = a_{\text{Venus}}\sqrt{1 - e_{\text{Venus}}^2} \approx 0{,}724\ \text{UA}.$$ **1p**

Problema 3

a)

$$\frac{1}{T_{\text{sinodic,M 2}}} = \frac{1}{T_{\text{sideral,P}}} - \frac{1}{T_{\text{sideral,M 2}}};$$

$$T_{\text{sideral,M 2}} = \frac{T_{\text{sideral,P}} \cdot T_{\text{sinodic,M 2}}}{T_{\text{sinodic,M 2}} - T_{\text{sideral,P}}} = \frac{1\ \text{an} \cdot 15\ \text{ani}}{15\ \text{ani} - 1\ \text{an}} = 1{,}07\ \text{ani}.$$

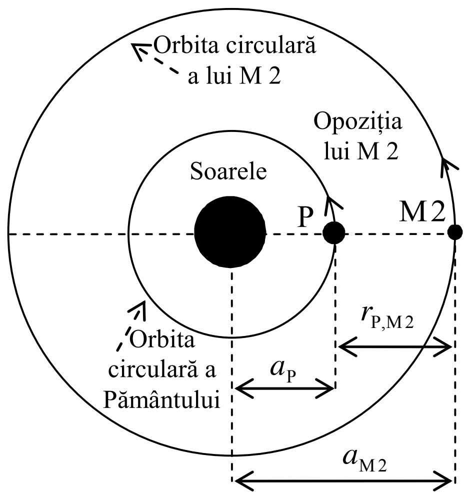


Fig.

$$\left(\frac{T_{\text{sideral,M}2}}{T_{\text{sideral,P}}}\right)^2=\left(\frac{a_{\text{M}2}}{a_{\text{P}}}\right)^3;$$

$$a_{\text{M}2}=a_{\text{P}}\cdot\sqrt[3]{\left(\frac{T_{\text{sideral,M}2}}{T_{\text{sideral,P}}}\right)^2}\approx 1{,}047\ \text{UA},\text{.....................................}\mathbf{2p}$$

b)

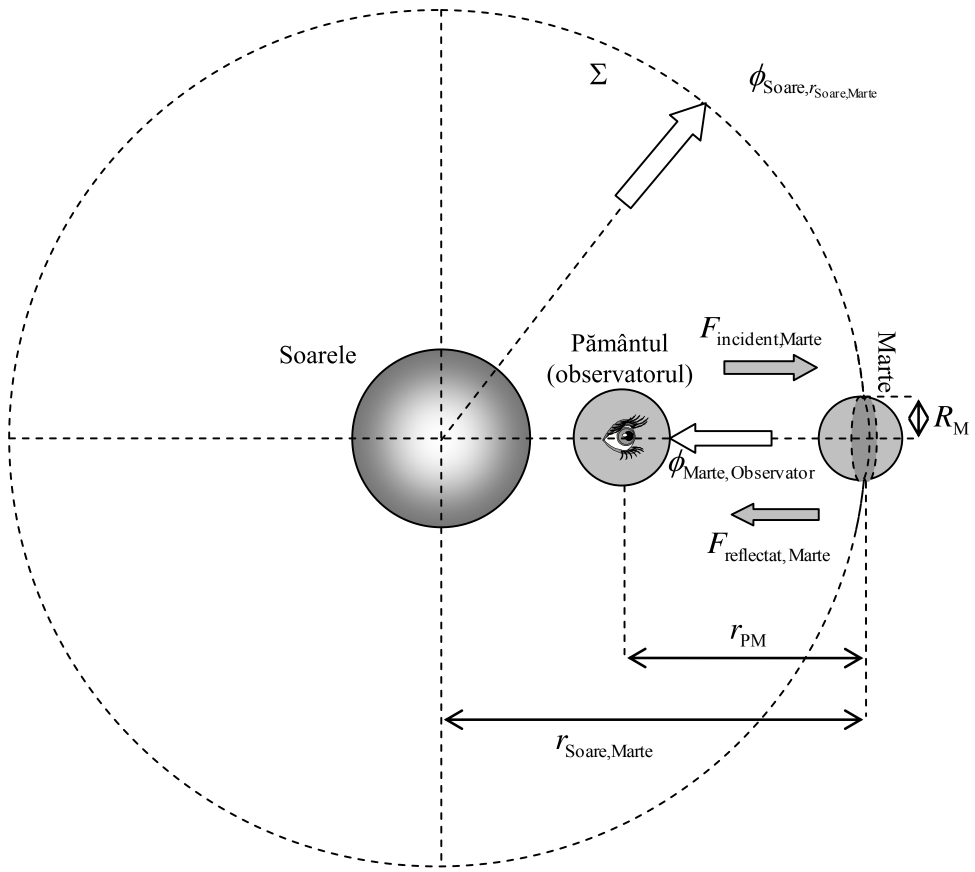


Fig.

$$\log\frac{\phi_{\text{Marte, Observator}}}{\phi_{\text{Marte 2,Observator}}}=-0{,}4\left(m_{\text{M}}-m_{\text{M}2}\right);$$

$$\log\frac{\alpha_{\text{M}}\cdot\dfrac{L_{\text{S}}}{4\pi r_{\text{S,M}}^2}\cdot\dfrac{\pi R_{\text{M}}^2}{2\pi r_{\text{P,M}}^2}}{\alpha_{\text{M}2}\cdot\dfrac{L_{\text{S}}}{4\pi r_{\text{S,M}2}^2}\cdot\dfrac{\pi R_{\text{M}2}^2}{2\pi r_{\text{P,M}2}^2}}=-0{,}4\left(m_{\text{M}}-m_{\text{M}2}\right);$$

$$\alpha_{\text{M}}=\alpha_{\text{M}2};\ R_{\text{M}}=R_{\text{M}2};$$

$$5\cdot\log\left(\frac{r_{\text{S,M}}}{r_{\text{S,M}2}}\cdot\frac{r_{\text{P,M}}}{r_{\text{P.M}2}}\right)=\left(m_{\text{M}}-m_{\text{M}2}\right);$$

$$5 \cdot \log(1,46 \cdot 11,15) = (m_{\mathrm{M}} - m_{\mathrm{M2}});$$

$$m_{\mathrm{M}} - m_{\mathrm{M2}} \approx 6^{\mathrm{m}};\ m_{\mathrm{M}} = -2^{\mathrm{m}};$$

$m_{\mathrm{M2}} = -8^{\mathrm{m}}$, ... **3p**

reprezentând magnitudinea aparentă a planetei “Marte 2”, văzută de pe Pământ.

c)

Atunci când Opoziția lui Marte se întâmplă la Periheliul orbitei sale, sau foarte aproape de acesta, avem de a face cu o Mare Opoziție a lui Marte, așa cum indică desenul din figura alăturată, distanța dintre Pământ și Marte fiind minimă posibilă (60 milioane km). Așa s-a întâmplat la 28 August 2003, aceasta fiind ultima Mare Opoziție a lui Marte.

Atunci când Opoziția lui Marte se întâmplă la Apheliul orbitei sale, sau foarte aproape de acesta, distanța dintre Pământ și Marte este maximă posibilă (100 milioane km).

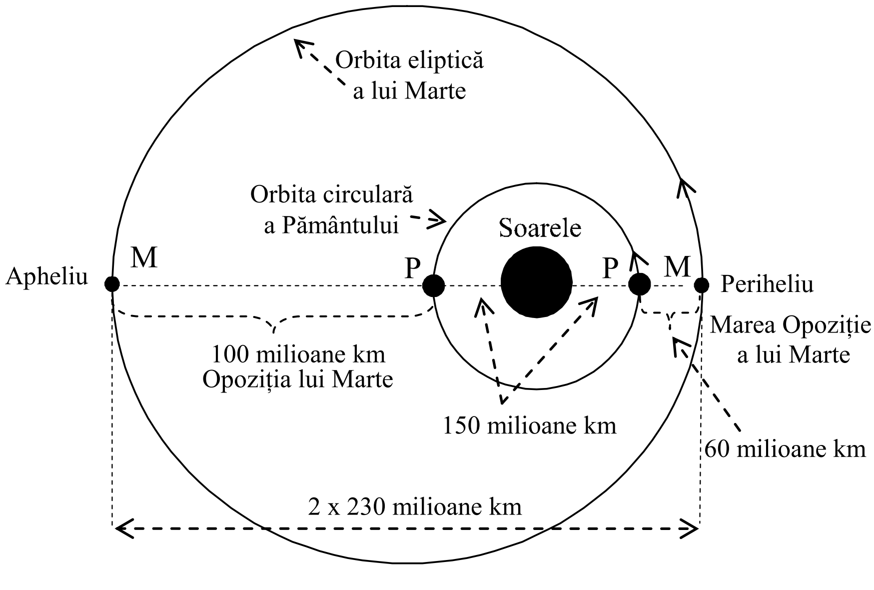


Fig.

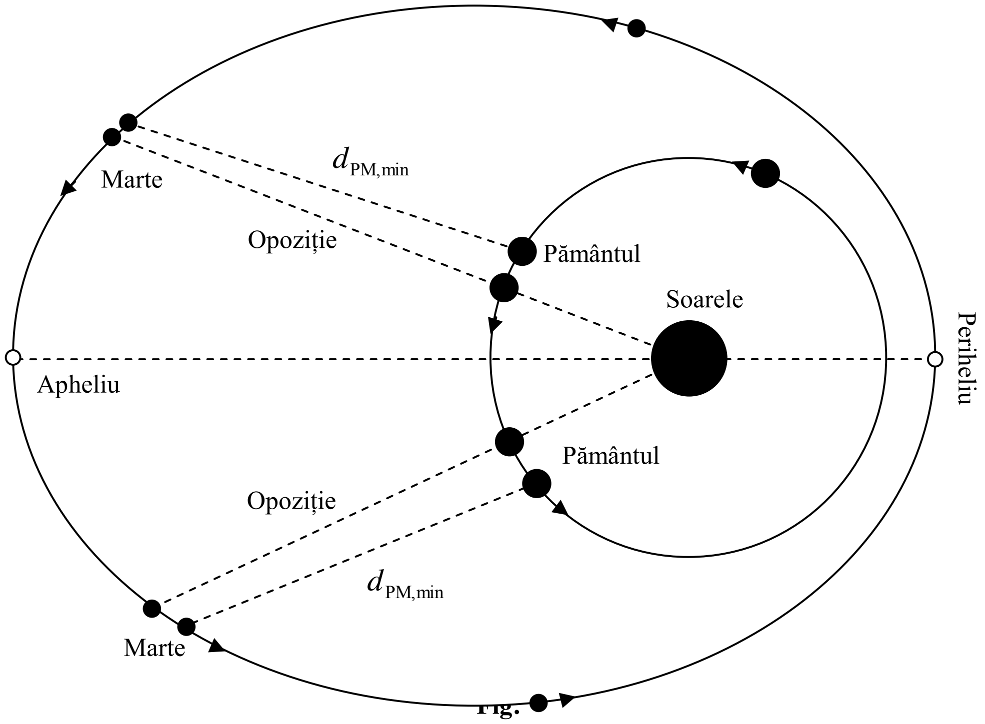
Tabelul de mai jos prezintă o listă a tuturor Opozițiilor lui Marte din 1955 și până în 2037. Din acest tabel rezultă că Pământul este relativ apropiat de Marte în anii 2001 și 2005, iar în 2003 Pământul este foarte aproape de Marte. Apoi, în anii 2020 și 2033, Pământul va fi din nou relativ apropiat de Marte, iar în anii 2018 și 2035 Pământul va fi din nou foarte aproape de Marte, ca și în 2003.

Opozițiile lui Marte, 1995 - 2037

Data Opoziției	**Data apropierii maxime**	**Distanța minimă** (UA/milioane mile)
12 Februarie 1995	11 Februarie 1995	0,67569/62,8
17 Martie 1997	20 Martie 1997	0,65938/61,3
24 Aprilie 1999	01 Mai 1999	0,57846/53,8
13 Iunie2001	**21 Iunie 2001**	**0,45017/41,8**
28 August 2003	***27 August 2003***	***0,37272/34,6***
07 Noiembrie2005	**30 Octombrie 2005**	**0,46406/43,1**
24 Decembrie 2007	18 Decembrie 2007	0,58935/54,8
29 Ianuarie 2010	27 Ianuarie 2010	0,66398/61,7
03 Martie 2012	05 Martie 2012	0,67368/62,6
08 Aprilie 2014	14 Aprilie 2014	0,61756/57,4
22 Mai 2016	**30 Mai 2016**	**0,50321/46,8**
27 Iulie 2018	***31 Iulie 2018***	***0,38496/35,8***
13 Octombrie 2020	**06 Octombrie 2020**	**0,41492/38,6**
08 Decembrie 2022	01 Decembrie 2022	0,54447/50,6
16 Ianuarie 2025	12 Ianuarie 2025	0,64228/59,7
19 Februarie 2027	20 Februarie 2027	0,67792/63,0
25 Martie 2029	29 Martie 2029	0,64722/60,2
04 Mai 2031	12 Mai 2031	0,55336/51,4
27 Iunie 2033	**05 Iulie 2033**	**0,42302/39,3**
15 Septembrie 2035	***11 Septembrie 2035***	***0,38041/35,4***
19 Noiembrie 2037	**11 Noiembrie 2037**	**0,49358/45,9**

În tabel sunt indicate două date: **data Opoziției**, când Pământul trece printre Marte și Soare, aliniindu-se cu aceștia; **data apropierii maxime** dintre Pământ și Marte, care este cu câteva zile mai devreme decât **data Opoziției**, când Marte se depărtează de Soare (apropiindu-se de Apheliu) și cu câteva zile mai târziu decât **data Opoziției**, când Marte se apropie de Soare (apropiindu-se de Periheliu), așa cum ilustrează desenul din figura alăturată.

Dacă data Opoziției este foarte aproape de Periheliu, atunci data apropierii maxime este aproximativ aceeași cu data Opoziției (așa cum s-a întâmplat în 2003). Pământul trece mai aproape de Marte, dacă data Opoziției este mai apropiată de Periheliu, așa cum se va întâmpla în anii 2018 și 2035.



Identificările Opozițiilor..3p

- Opozițiile marcate cu BOLD ITALIC se produc atunci când distanța dintre Pământ și Marte este minimă, ceea ce se întâmplă când Marte este la Periheliu sau foarte aproape de acesta.

- Opozițiile marcate cu BOLD DREPT se produc atunci când distanța dintre Pământ și Marte este mică, ceea ce se întâmplă când Marte este în apropierea Periheliului.

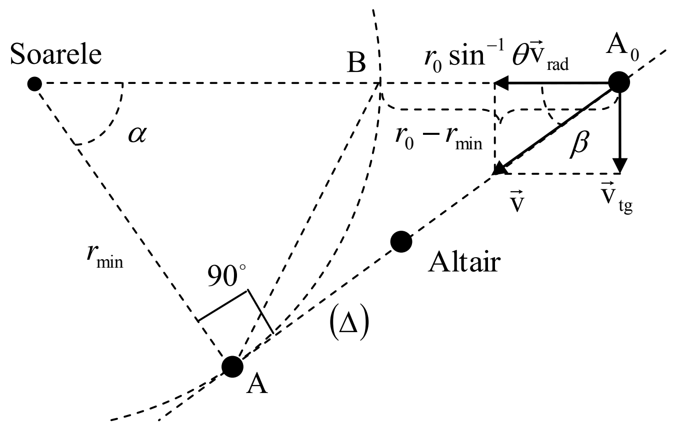
- Opozițiile pentru care distanțele dintre Pământ și Marte sunt maxime se produc atunci când Marte este la Apheliului.

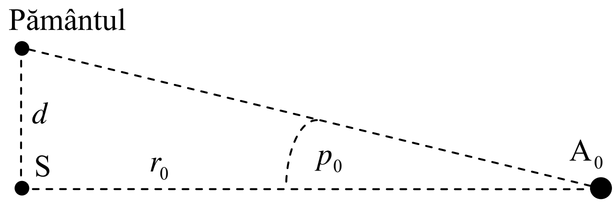
- Opozițiile pentru care distanțele dintre Pământ și Marte sunt apropiate de valorile maxime se produc atunci când Marte este aproape de Apheliul orbitei sale.

d)

$$\frac{p_{\mathrm{e,S}}}{p_{\mathrm{e,M}}} = (1-e)\left(\frac{T_{\mathrm{M}}}{T_{\mathrm{P}}}\right)^{2/3} - 1 \approx 1{,}54 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathbf{2p}$$

Problema 4
A.





$$\frac{r_{min}}{r_0} = \sqrt{1 - \frac{v_{rad}^2}{v_{rad}^2 + (\mu r_0)^2}};$$

$$\log \frac{E_0}{E} = -0{,}4(m_0 - m);$$

$$m = m_0 + 5 \cdot \log \frac{\frac{\mu D}{p_0}}{\sqrt{v_{rad}^2 + \left(\frac{\mu D}{p_0}\right)^2}};$$

$m = m_0 + 5^m \cdot (0{,}3) = 0{,}89^m - 1{,}5^m = -0{,}61^m$; **2p**

$$r_0 = \frac{D}{p_0} = \frac{15 \cdot 10^7 \text{ km}}{10^{-6}} = 15 \cdot 10^{13};$$

$$r_{\min} = r_0 \cdot \frac{\mathrm{v_{tg}}}{\sqrt{\mathrm{v_{rad}^2} + \mathrm{v_{tg}^2}}} = 15 \cdot 10^{13}\ \mathrm{km} \cdot \frac{1}{2} = 7{,}5 \cdot 10^{13}\ \mathrm{km}; \text{..........................}\mathbf{2p}$$

$$r_0 - r_{\min} = \mathrm{v_{rad}}\,\tau;$$

$$\tau = \frac{r_0 - r_{\min}}{\mathrm{v_{rad}}} \approx 3 \cdot 10^{12}\ \mathrm{s} \approx 95130\,\text{ani}\text{..............................}\mathbf{1p}$$

B.

$$\log \frac{E_\mathrm{S}}{E_\sigma} = -0{,}4\left(m_\mathrm{S} - m_\sigma\right),$$

$$k_\mathrm{S} = E_\mathrm{S} \quad ; \; k_\sigma = E_\sigma$$

$$k_\sigma = k_\mathrm{S} \cdot 10^{-\left(m_\sigma - m_\mathrm{S}\right)/2{,}5};\; k_\mathrm{S} = 1{,}37\ \mathrm{kW/m^2};$$

$$m_\sigma = 1^\mathrm{m};\; m_\mathrm{S} = -26{,}8^\mathrm{m};\; k_\sigma \approx 10^{-8}\ \frac{\mathrm{W}}{\mathrm{m^2}}.$$

$$W_\mathrm{total} = k_\sigma \cdot S \cdot t = 10^{-8}\ \frac{\mathrm{J}}{\mathrm{m^2 \cdot s}} \cdot 1000\ \mathrm{m^2} \cdot 3 \cdot 10^8\ \mathrm{s} = 3000\ \mathrm{J},$$

$$W = \eta \cdot W_\mathrm{total} = 300\ \mathrm{J}.$$

$$M = \rho V = 2 \cdot 10^6\ \mathrm{kg}.$$

$$Mc\Delta\theta = W;\; \Delta\theta = \frac{W}{Mc} = \frac{300\ \mathrm{J}}{2 \cdot 10^6\ \mathrm{kg} \cdot 4200\ \frac{\mathrm{J}}{\mathrm{kg \cdot K}}} \approx 3{,}5 \cdot 10^{-8}\ \mathrm{K}\text{..................................}\mathbf{2p}$$

C.

$$m_\mathrm{Soare} = m_\mathrm{stea} - 5 \cdot \log \frac{\delta_\mathrm{Soare}}{\delta_\mathrm{stea}} - 10 \cdot \log \frac{T_\mathrm{Soare}}{T_\mathrm{stea}}\text{..}\mathbf{2p}$$

$$\overline{m}_\mathrm{Soare} = -25{,}195^\mathrm{m}\text{...}\mathbf{1p}$$